An Approach to Capture Stable Models with Classical Ones

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BACKGROUND AND MOTIVATION

➢ Normal logic programs (under stable model semantics) vs. sets of clauses (under classical models).

➢ If P is *positive-order-consistent* (or *tight*), then the stable models of P and the classical models of $\mathrm{Comp}(P)$ coincide.

Y. Babovich, E. Erdem, and V. Lifschitz [NMR Workshop, 2000]: *Fages' Theorem and Answer Set Programming*.

➢ Removing positive literals from the rule bodies is not easy.

T. Janhunen [CL 2000]: *Comparing the Expressive Powers of Some Syntactically Restricted Classes of Logic Programs*.

➢ *Partial evaluation* takes an exponential space in the worst case.

OUTLINE OF THE TALK

1. Classes of Logic Programs
2. Properties of Translation Functions
3. Classification Method
4. Non-modular Translation
5. Optimizations in Sight
6. Implementation and Experiments
7. Related Work
8. Conclusions

1. CLASSES OF LOGIC PROGRAMS

Definition: *A propositional logic program P is a set of expressions (such as rules, clauses or sentences) built of propositional atoms.*

➢ We let $\mathrm{Hb}(P)$ stand for the Herbrand base of a logic program P.

➢ General requirements for a class $\mathcal{C}$ of logic programs:

1. Closure under unions: if $P \in \mathcal{C}$ and $Q \in \mathcal{C}$, then $P \cup Q \in \mathcal{C}$.
2. There is a semantical operator $\mathrm{Sem}_{\mathcal{C}}$ that maps any $P \in \mathcal{C}$ to $\mathrm{Sem}_{\mathcal{C}}(P) \subseteq \mathbf{2}^{\mathrm{Hb}(P)}$, i.e., the **set of models** of P.

$\Longrightarrow$ The class of sets of propositional clauses under classical models ($\mathcal{SAT}$) can be viewed as a class of logic programs.

Some Syntactic Subclasses of Normal Programs

- The representatives of the class of normal logic programs $\mathcal{N}$ that consist of rules of the form $h \leftarrow b_1, \ldots, b_n, {\sim}c_1, \ldots, {\sim}c_m$.
- The number of positive body literals n is constrained as follows:
  1. The class of **atomic programs** $\mathcal{A}$ ($n = 0$ for every rule).
  2. The class of **unary programs** $\mathcal{U}$ ($n \leq 1$ for every rule).
  3. The class of **binary programs** $\mathcal{B}$ ($n \leq 2$ for every rule).

  ☞ $\mathcal{A} \subset \mathcal{U} \subset \mathcal{B} \subset \mathcal{N}$.
- **Strictly unary** ($n = 1$), **strictly binary** ($n = 2$) and **non-binary** $n > 2$ rules are also distinguished.
- Our criteria for classes of logic programs are met (the semantics of the programs is determined by stable models in each class).

2. PROPERTIES OF TRANSLATION FUNCTIONS

Definition: *A translation function* $\mathrm{Tr} : \mathcal{C} \to \mathcal{C}'$ *is* **PFM** $\iff$ Tr *is*

- **polynomial (P)**,

  The time required to compute $\mathrm{Tr}(P)$ *is polynomial in* $||P||$.
- **faithful (F)**, *and*
  1. *For all programs* $P \in \mathcal{C}$, $\mathrm{Hb}(P) \subseteq \mathrm{Hb}(\mathrm{Tr}(P))$.
  2. *The models/interpretations in* $\mathrm{Sem}_{\mathcal{C}}(P)$ *and* $\mathrm{Sem}_{\mathcal{C}'}(\mathrm{Tr}(P))$ *are in a bijective relationship and coincide up to* $\mathrm{Hb}(P)$.
- **modular (M)**.
  1. *For every* $P \in \mathcal{C}$ *and* $Q \in \mathcal{C}$, $\mathrm{Tr}(P \cup Q) = \mathrm{Tr}(P) \cup \mathrm{Tr}(Q)$.
  2. *If* $\mathcal{C}' \subset \mathcal{C}$, *then* $\mathrm{Tr}(P') = P'$ *for every* $P' \in \mathcal{C}'$.

3. CLASSIFICATION METHOD

Given two classes $\mathcal{C}$ and $\mathcal{C}'$ of programs, the goal is to establish either

- $\mathcal{C} \underset{\text{PFM}}{\longrightarrow} \mathcal{C}'$ (there is a PFM translation function $\mathrm{Tr} : \mathcal{C} \to \mathcal{C}'$), **or**
- $\mathcal{C} \underset{\text{PFM}}{\not\longrightarrow} \mathcal{C}'$ (such a translation function does not exist).

These relations induce further relations for classes of logic programs:

Relation	Definition	Explanation
$\mathcal{C} \underset{\text{PFM}}{\Longrightarrow} \mathcal{C}'$	$\mathcal{C} \underset{\text{PFM}}{\longrightarrow} \mathcal{C}'$ and $\mathcal{C}' \underset{\text{PFM}}{\not\longrightarrow} \mathcal{C}$	*less expressive*
$\mathcal{C} \underset{\text{PFM}}{\longleftrightarrow} \mathcal{C}'$	$\mathcal{C} \underset{\text{PFM}}{\longrightarrow} \mathcal{C}'$ and $\mathcal{C}' \underset{\text{PFM}}{\longrightarrow} \mathcal{C}$	*equally expressive*
$\mathcal{C} \underset{\text{PFM}}{\not\longleftrightarrow} \mathcal{C}'$	$\mathcal{C} \underset{\text{PFM}}{\not\longrightarrow} \mathcal{C}'$ and $\mathcal{C}' \underset{\text{PFM}}{\not\longrightarrow} \mathcal{C}$	*mutually incomparable*

☞ Classes can be ordered on the basis of their expressive power.

Expressive Power Hierarchy

- It is straightforward to encode propositional satisfiability problems as an (atomic) normal programs: $\mathcal{SAT} \underset{\text{PFM}}{\longrightarrow} \mathcal{A}$.
- $\mathcal{A} \subset \mathcal{U} \subset \mathcal{B} \subset \mathcal{N} \implies \mathcal{A} \underset{\text{PFM}}{\longrightarrow} \mathcal{U} \underset{\text{PFM}}{\longrightarrow} \mathcal{B} \underset{\text{PFM}}{\longrightarrow} \mathcal{N}$ (by identity).
- Non-binary rules can be translated away: $\mathcal{N} \underset{\text{PFM}}{\longrightarrow} \mathcal{B}$.
- Binary and unary rules cannot be translated away in a **faithful and modular** way: $\mathcal{B} \underset{\text{FM}}{\not\longrightarrow} \mathcal{U}$ and $\mathcal{U} \underset{\text{FM}}{\not\longrightarrow} \mathcal{A}$.
- Due to non-monotonicity $\mathcal{A} \underset{\text{FM}}{\not\longrightarrow} \mathcal{SAT}$.
- The resulting hierarchy of the five classes under consideration:

$$\mathcal{SAT} \underset{\text{PFM}}{\Longrightarrow} \mathcal{A} \underset{\text{PFM}}{\Longrightarrow} \mathcal{U} \underset{\text{PFM}}{\Longrightarrow} \mathcal{B} \underset{\text{PFM}}{\longleftrightarrow} \mathcal{N}$$

4. NON-MODULAR TRANSLATION

A normal logic program P is translated into an **atomic** one $\mathrm{Tr}(P)$.

➢ The goal is a polynomial and faithful translation such that $||\mathrm{Tr}(P)||$ is of order $||P|| \times \log_2 |\mathrm{Hb}(P)|$.

➢ As a characteristic of a stable model M of P, the numbers

$$\mathrm{level}(a) = \begin{cases} \min\limits_{\substack{\mathrm{head}(r)=a \\ M \models \mathrm{body}(r)}} \mathrm{level}(r), & \text{if } M \models a. \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathrm{level}(r) = \begin{cases} \max\limits_{b \in \mathrm{body}^+(r)} \mathrm{level}(b) + 1, & \text{if } \mathrm{body}^+(r) \neq \emptyset. \\ 0 & \text{otherwise.} \end{cases}$$

are uniquely defined for any $a \in \mathrm{Hb}(P)$ and $r \in P$.

➢ Level numbers lead to a characterization of stable models:

Theorem:

1. If M is a *stable model* of P, then M is a *supported model* of P and there is a level numbering (as described above) for atoms $a \in \mathrm{Hb}(P)$ and rules $r \in P$.
2. If M is a *supported model* of P and there is a level numbering (as described above) for atoms $a \in \mathrm{Hb}(P)$ and rules $r \in P$, then M is *stable model* of P.

➢ This suggests a translation that consists of two independent parts:

1. The first part captures supported models of P.
2. The second part checks if one can assign a level numbering (as described above) for atoms $a \in \mathrm{Hb}(P)$ and $r \in P$.

Translation of Atoms

An atom $a \in \mathrm{Hb}(P)$ is translated as follows:

➢ The truth value of a is chosen:

$$a \leftarrow \sim\overline{a}. \quad \overline{a} \leftarrow \sim a.$$

➢ Let $N = \lceil \log_2(|\mathrm{Hb}(P)| + 1) \rceil$ and let us introduce a counter (a vector of atoms) $a_{\mathrm{cnt}} = a_1 \ldots a_N$ and $\overline{a_{\mathrm{cnt}}} = \overline{a_1} \ldots \overline{a_N}$.

➢ The value of a_{cnt} is chosen: for each $0 < i \leq N$,

$$a_i \leftarrow \sim\overline{a_i}. \quad \overline{a_i} \leftarrow \sim a_i.$$

➢ Similarly, we need to define another counter a_{succ} that takes the value of a_{cnt} incremented by one: subprogram $\mathrm{SUCC}(a_{\mathrm{cnt}}, a_{\mathrm{succ}})$.

Translation of Rules

A new atom r (and the complement $\overline{r}$) is introduced for each rule $a \leftarrow b_1, \ldots, b_n, \sim c_1, \ldots, \sim c_m$ in P.

Rules that are needed to capture *supported models* of P:

➢ Evaluate the body of the rule:

$$r \leftarrow \sim\overline{b}_1, \ldots, \sim\overline{b}_n, \sim c_1, \ldots, \sim c_m. \quad \overline{r} \leftarrow \sim r.$$

➢ Every rule must be satisfied:

$$\bot \leftarrow \sim a, \sim\overline{r}.$$

plus $\top \leftarrow \sim\bot$ and $f \leftarrow \sim f, \sim\top$ where $\top$, $\bot$ and f are new.

➢ Every atom must have a supporting rule:

$$a_{\mathrm{supp}} \leftarrow \sim\overline{r}. \quad \bot \leftarrow \sim\overline{a}, \sim a_{\mathrm{supp}}.$$

Translation of Rules (Continued)

- A counter is introduced for each rule:

  $$r_{\mathrm{cnt}} = r_1 \ldots r_N \text{ and } \overline{r_{\mathrm{cnt}}} = \overline{r_1} \ldots \overline{r_N}.$$

  Thus we need $r_i \leftarrow \sim\overline{r_i}$ and $\overline{r_i} \leftarrow \sim r_i$ for each $0 < i \leq N$.
- The value of r_{cnt} is supposed to be $\mathrm{level}(r)$ in binary.
- If $n > 0$, we need for each $b \in \{b_1, \ldots, b_n\}$ subprograms $\mathrm{LTHAN}(r_{\mathrm{cnt}}, b_{\mathrm{succ}})$ and $\mathrm{EQ}(r_{\mathrm{cnt}}, b_{\mathrm{succ}})$ in addition to

  $$\perp \leftarrow \sim\overline{\mathrm{lthan}(r_{\mathrm{cnt}}, b_{\mathrm{succ}})}.$$

  $$r_{\mathrm{eq}} \leftarrow \sim\overline{\mathrm{eq}(r_{\mathrm{cnt}}, b_{\mathrm{succ}})}. \quad \perp \leftarrow \sim r_{\mathrm{eq}}.$$
- If $n = 0$, we need the subprogram $\mathrm{ZERO}(r_{\mathrm{cnt}})$ and the rules

  $$\perp \leftarrow \sim\mathrm{zero}(r_{\mathrm{cnt}}).$$

Translation of Atoms (Continued)

- The value of a_{cnt} is supposed to be $\mathrm{level}(a)$ in binary.
- For each rule r having a as head, we need the subprograms $\mathrm{LTHAN}(r_{\mathrm{cnt}}, a_{\mathrm{cnt}})$ and $\mathrm{EQ}(r_{\mathrm{cnt}}, a_{\mathrm{cnt}})$ plus the following rules:

  $$\perp \leftarrow \sim\overline{r}, \sim\overline{\mathrm{lthan}(r_{\mathrm{cnt}}, a_{\mathrm{cnt}})}.$$

  $$a_{\mathrm{eq}} \leftarrow \sim\overline{r}, \sim\overline{\mathrm{eq}(r_{\mathrm{cnt}}, a_{\mathrm{cnt}})}. \quad \perp \leftarrow \sim\overline{a}, \sim a_{\mathrm{eq}}.$$

  Note that $\sim\overline{a}$ is not needed when $\sim\overline{r}$ is present.
- For every $a \in \mathrm{Hb}(P)$, we need $\mathrm{ZERO}(a_{\mathrm{cnt}})$ and the rules

  $$\perp \leftarrow \sim a, \sim\mathrm{zero}(a_{\mathrm{cnt}}).$$

5. OPTIMIZATIONS IN SIGHT

- For certain graph problems, the preceding technique yields translations of size $30 \times ||P|| \times \log_2 |\mathrm{Hb}(P)|$ approximately.
- Counters can be avoided when simple rules are translated:
  1. Atomic r: $h \leftarrow \sim c_1, \ldots, \sim c_m$: $\perp \leftarrow \sim\overline{r}, \sim\mathrm{zero}(h_{\mathrm{cnt}})$.
  2. Unary r: $h \leftarrow b, \sim c_1, \ldots, \sim c_m$:
     $\perp \leftarrow \sim\overline{r}, \sim\overline{\mathrm{lthan}(h_{\mathrm{cnt}}, b_{\mathrm{succ}})}$. $\quad h_{\mathrm{eq}} \leftarrow \sim\overline{\mathrm{eq}(h_{\mathrm{cnt}}, b_{\mathrm{succ}})}$.
  3. Binary r: $h \leftarrow b, b', \sim c_1, \ldots, \sim c_m$: the rules

     $$h \leftarrow b, \sim c_1, \ldots, \sim c_m, \sim\mathrm{lthan}(b_{\mathrm{succ}}, b'_{\mathrm{succ}}).$$
     $$h \leftarrow b', \sim c_1, \ldots, \sim c_m, \sim\overline{\mathrm{lthan}(b_{\mathrm{succ}}, b'_{\mathrm{succ}})}.$$

     are translated like unary rules above.
- The general scheme applies to non-binary rules (if such exist).

Exploiting Dependency Information

- It is possible to localize the translation to strongly connected components (SCCs) of the program.
- Only **positive dependencies** are taken into account.
- A SCC may contain several **loops** $a_1 \rightsquigarrow a_2 \rightsquigarrow \ldots \rightsquigarrow a_n = a_1$.
  Example: $P = \{\ a \leftarrow \sim b. \ \ b \leftarrow \sim a.\ \}$ is free of loops.
- Any rule $h \leftarrow b_1, \ldots, b_n, \sim c_1, \ldots, \sim c_m$ whose head h is not involved in any loop is easily made atomic:

  $$h \leftarrow \sim\overline{b_1}, \ldots, \sim\overline{b_n}, \sim c_1, \ldots, \sim c_m.$$

  $$\overline{b_1} \leftarrow \sim b_1. \quad \ldots \quad \overline{b_n} \leftarrow \sim b_n.$$
- Rules forming a self-loop ($h \in \{b_1, \ldots, b_n\}$) can/must be deleted!

6. IMPLEMENTATION AND EXPERIMENTS

➢ We have implemented two translator programs:

1. LP2ATOMIC transforms an SMODELS program into atomic one.
2. LP2SAT produces Clark's completion in DIMACS format.

Only basic rules and compute statements are covered!

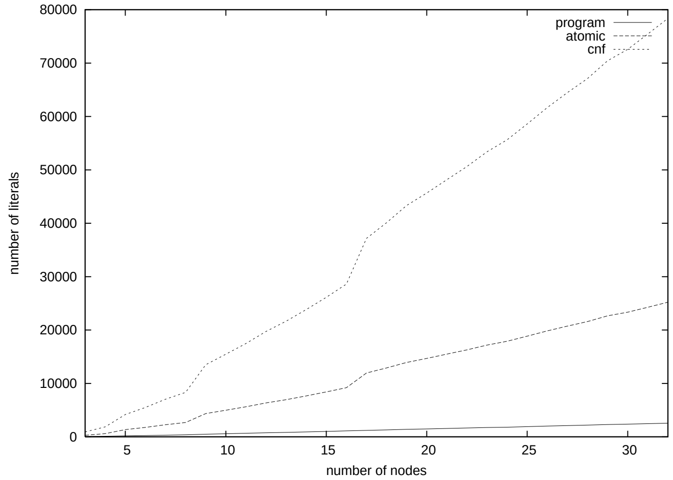
➢ The problem of finding a Hamiltonian circuit for a random planar graph (Niemelä's encoding) is used as a benchmark to compare:

- SMODELS (version 2.26) vs.
- LP2ATOMIC + LP2SAT + ZCAFF (version 2001.2.17).

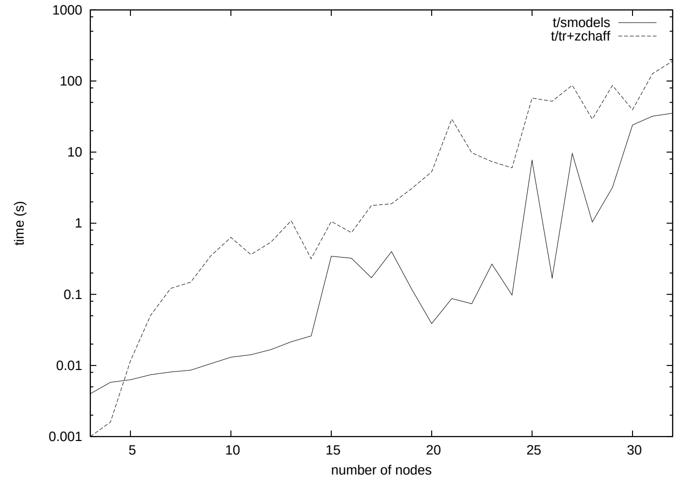
➢ The number of nodes was varied from 3 to 32 (100 instances).

➢ Tests were run on a 451 MHz Pentium III PC running under Linux operating system (kernel version 2.2.19).

Size of the Translation



Time Required for Computing One Model



7. RELATED WORK

➢ R. Ben-Eliyahu and R. Dechter [AMAI, 1994]: *Propositional semantics for disjunctive logic programs.*

- Binary numbers are not used $\Longrightarrow$ at least quadratic encoding.
- The stable models of P and the classical models of $\mathrm{Tr}_{\mathrm{ED}}(P)$ are not in a bijective relationship.

Example: Let $P = \{a \leftarrow b, c.\ \ b \leftarrow d.\ \ c \leftarrow d.\ \ d \leftarrow \sim e.\ \ d \leftarrow a\}$. The atoms b and c in the unique stable model $M = \{a, b, c, d\}$ can be ordered in two different ways (in a **total ordering**).

➢ Y. Babovich, E. Erdem, and V. Lifschitz [NMR Workshop, 2000]: *Fages' Theorem and Answer Set Programming.*

- Programs containing loops are not (necessarily) covered.
- Tightness is based on a different numbering of atoms.

➢ U. Egly, et al. [AAAI, 2001]: *Computing Stable Models with Quantified Boolean Formulas: Some Experimental Results*.

- In this approach, disjunctive stable models are captured with quantified Boolean formulas $\exists p_1 \ldots \exists p_n \forall q_1 \ldots \forall q_m \, \phi$.
- In particular, the minimality requirement of disjunctive stable models is easy to express in terms of such primitives.
- However, the computation of stable models is easier in the case of normal logic programs.

➢ F. Lin and Y. Zhao [AAAI, 2002]: *ASSAT: Computing Answer Sets of a Logic Program by SAT Solvers*.

- The idea is to extend $\mathrm{Comp}(P)$ gradually with *loop formulas*.
- In the worst case, there is an exponential number of loops (Hamiltonian paths for complete graphs).

8. CONCLUSIONS

➢ A PF translation was established: $\mathcal{N} \overset{\rightarrow}{\scriptstyle\mathrm{PF}} \mathcal{A}$, $\mathcal{B} \overset{\rightarrow}{\scriptstyle\mathrm{PF}} \mathcal{A}$, and $\mathcal{U} \overset{\rightarrow}{\scriptstyle\mathrm{PF}} \mathcal{A}$.

➢ Clark's completion is sufficient for $\mathcal{A} \overset{\rightarrow}{\scriptstyle\mathrm{PF}} \mathcal{SAT}$.

➢ **Theorem:** $\mathcal{SAT} \overset{\leftrightarrow}{\scriptstyle\mathrm{PF}} \mathcal{A} \overset{\leftrightarrow}{\scriptstyle\mathrm{PF}} \mathcal{U} \overset{\leftrightarrow}{\scriptstyle\mathrm{PF}} \mathcal{B} \overset{\leftrightarrow}{\scriptstyle\mathrm{PF}} \mathcal{N}$.

➢ Distinctive features of the counter-based approach:

1. bijective relationship of models and
2. $||\mathrm{Tr}(P)||$ is of order $\displaystyle\sum_{S \text{ is a SCC of } P} ||P_S|| \times \log_2(|S| + 1)$.

➢ Transitive closure can be properly captured with classical models.

➢ Experimental results are promising, but further optimizations should be pursued for in order to really compete with SMODELS.