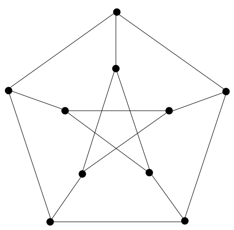
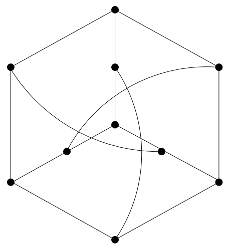
Introduction to Discrete Mathematics, Fall 2000
Problem Set 6, 31 October – 2 November

1. Draw a picture of the complete bipartite graph $K_{2,4}$ and write down its adjacency matrix. Describe in general terms the adjacency matrices of graphs K_n, $K_{m,n}$ and C_n.

2. Prove that the two graphs in the following picture are isomorphic. (The picture shows two layouts of the *Petersen graph*, which is an often encountered example or counterexample in many graph-theoretic constructions.)

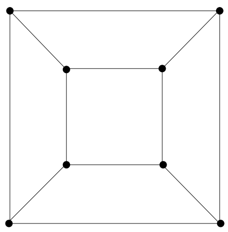


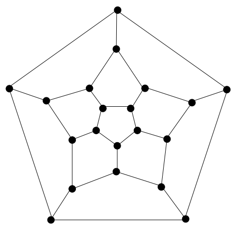


3. Draw pictures of all the non-isomorphic 4-vertex (undirected, simple) graphs. How long would it approximately take to enumerate all the 15-vertex graphs on a computer capable of enumerating one graph per nanosecond?

4. Prove that if an apartment has only one door out, then at least one of its rooms must have an odd number of doors.

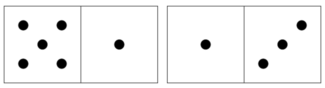
5. It was observed by Sir William Rowan Hamilton in 1856 that every graph generated by the vertices and edges of a regular solid contains a Hamiltonian cycle. Verify his observation for the following graphs that correspond to the cube and the regular dodecahedron (regular 12-sided solid).





6. Let $G = (V, E)$ be an (undirected) graph, all of whose vertices are of even degree. Prove that the edges of G can be oriented so that in the resulting directed graph the in- and outdegrees of each vertex (i.e. the number of incoming and outgoing arcs at that vertex) are equal.

7. A piece in the game of dominoes has the shape of a rectangle divided into two squares, each of which contains from zero to six "eyes". If one leaves out the pieces whose sides contain twice the same number of eyes, and considers the orientation of a piece inessential, one observes that the pieces correspond exactly to two-element sets $\{i, j\}$, where $i, j = 0, \ldots, 6$, $i \neq j$. Prove that the $\binom{7}{2} = 21$ domino pieces thus obtained can be ordered into a closed ring that respects the rules of the game, i.e. where the number of eyes in two adjacent squares are always equal. As an example, the following picture shows a legal placing of the pieces $\{1, 5\}$ ja $\{1, 3\}$.



(*Hint:* Consider the graph K_7.)