

FPGA Design of Self-certified Signature Verification on Koblitz Curves

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Outline

1 Preliminaries

- Introduction
- Koblitz curves
- Signatures

2 Algorithms and Implementation

- Point multiplication
- Precomputation
- Implementation

3 Results and Discussion

- Results on an FPGA
- Conclusions and future work

Introduction

- Packet Level Authentication (PLA)¹
 - Enormous speed requirements!
 - **Elliptic curve cryptography** because short signatures and fast performance are needed
 - **Koblitz curve, NIST K-163**, used to maximize speed
 - **Self-certified ID based signatures** because they are short and computationally less complex

¹See <http://www.tcs.hut.fi/Software/PLA/new/index.shtml>

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 - **Self-certified ID based signatures** because they are short and computationally less complex
- Development in FPGA technology
 - Growth in resources enables massive parallelization
 - Point multiplication times $< 100 \mu\text{s}$ have been reported
- We focus on **maximizing operations per second** instead of minimizing computation time of a single operation

¹See <http://www.tcs.hut.fi/Software/PLA/new/index.shtml>

Koblitz curves

- Koblitz curves have the form

$$E_K : y^2 + xy = x^3 + ax^2 + 1$$

- If $P = (x, y)$ is a point on E_K , then its **Frobenius endomorphism**, $\phi(P) = (x^2, y^2)$, is also on E_K .
- Very efficient point multiplication
 - Integer presented in τ -adic non-adjacent form (NAF)²
 - Point doublings replaced by Frobenius maps
 - Only $m/3$ point additions

²Solinas, Des. Codes Cryptogr. 19(2-3), 2000

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A signature is verified by computing:

$$W_A = \text{DECOMPRESS}(r_A - \text{HASH}(ID_A), b_A) - r_A W_D, \text{ and}$$

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$$dG + c(uG) - cr_A W_D = k_1 P_1 + k_2 P_2 + k_3 P_3$$

Point multiplication

$$Q = k_1 P_1 + k_2 P_2 + k_3 P_3$$

- Shamir's trick \Rightarrow 3-term double-and-add algorithm
- 3-term τ -adic joint sparse form³

Simplified algorithm

- ① Precompute all possible combinations
 $R_{k_1, k_2, k_3} = k_{1,j} P_1 + k_{2,j} P_2 + k_{3,j} P_3$
- ② Perform $\phi(P)$ for all bits
- ③ If $k_{1,j}, k_{2,j}, k_{3,j} \neq 000$, add R_{k_1, k_2, k_3} to Q using mixed coordinate point addition^a

^aAl-Daoud et al. IEEE Tran. Comp. 51(8), 2002

³Brumley, ICICS 2006, LNCS 4307

Precomputed points

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- Pairs (R_k, R_{k+1}) are computed so that
 - $R_k = R_i + R_j$, and
 - $R_{k+1} = R_i - R_j$
- Unified point addition and subtraction:
$$(R_k, R_{k+1}) \leftarrow R_i \pm R_j$$

Unified point addition and subtraction

Point addition

$$(x_3, y_3) = (x_1, y_1) + (x_2, y_2)$$

$$\lambda = \frac{y_1 + y_2}{x_1 + x_2}$$

$$x_3 = \lambda^2 + \lambda + x_1 + x_2 + a$$

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Point subtraction

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- Inversion is the same⁴
- Some additions can be saved by rearranging operations
- Total cost reduces from $2I + 4M + 2S + 17A$ to $I + 4M + 2S + 14A$

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Montgomery's trick

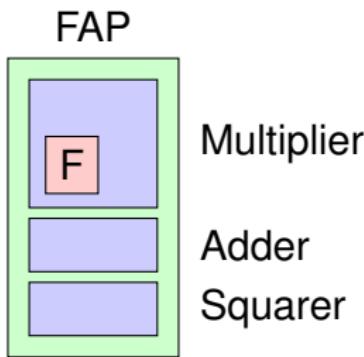
Method	Cost	$I = 9M$
Naïve	$10(I + 2M + S + 8A) + 5A$	110M
Unified	$5(I + 4M + 2S + 14A)$	65M

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Unified + Montgomery	$I + 17M + 2S + 9A + 5(4M + 2S + 14A)$	46M

- Trades inversions to multiplications
- $1/x_1$ and $1/x_2$ computed so that $1/x_1 = x_2/(x_1 x_2)$ and $1/x_2 = x_1/(x_1 x_2)$
- n inversions computed with $3(n - 1)$ multiplications and 1 inversion

Architecture

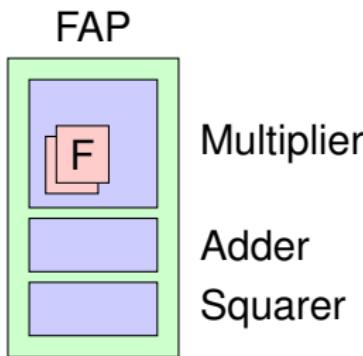


Massey-Omura multiplier

- Bit-serial, only one F -block
- Latency: $m + c + 1$ clock cycles

Area →
Time →
Ops →

Architecture

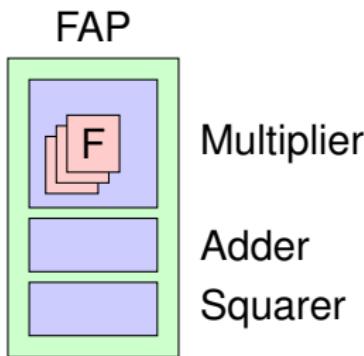


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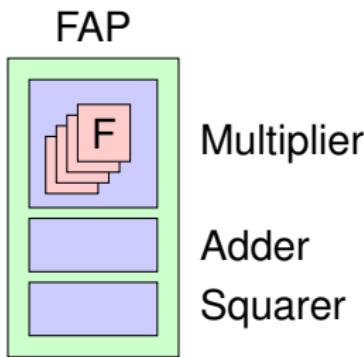


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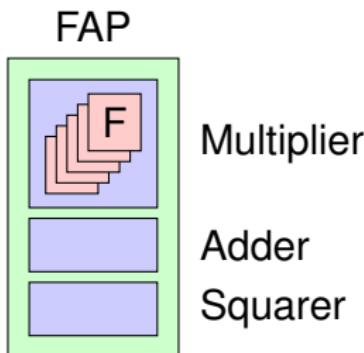


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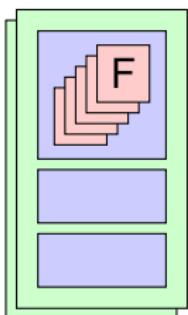
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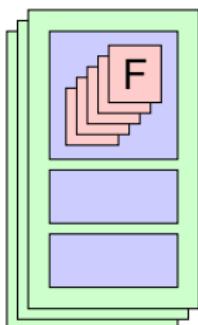


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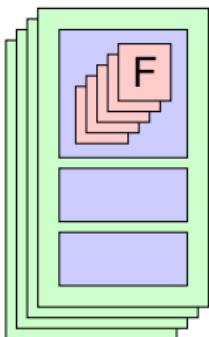
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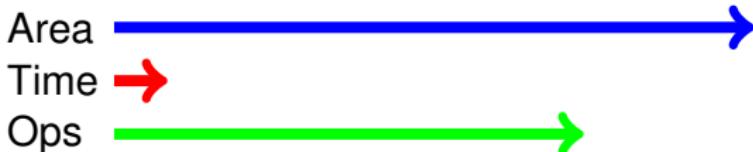
A horizontal blue arrow points to the right, indicating increasing area. A red arrow points downwards, indicating increasing time. A green arrow points to the right, indicating increasing operations.

Architecture

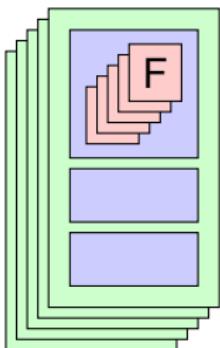


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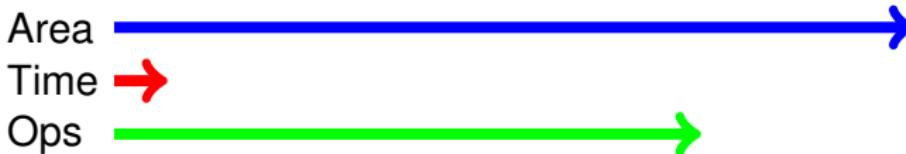


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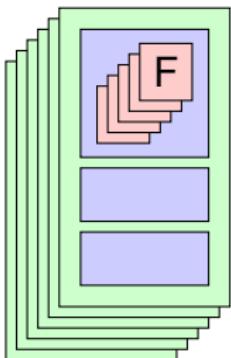


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Architecture

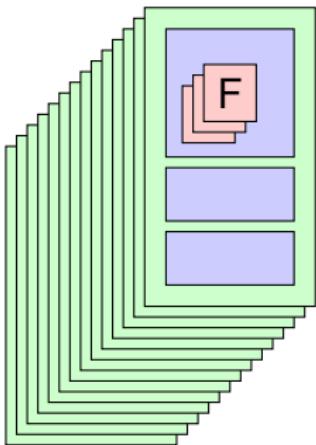


Massey-Omura multiplier

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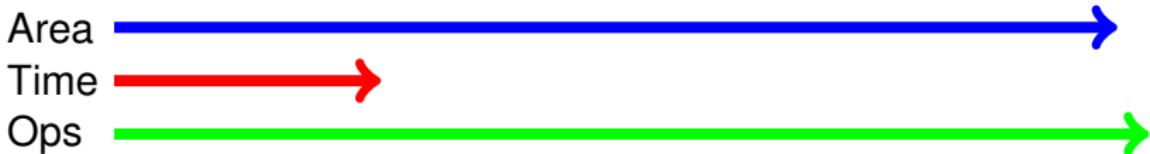


Architecture

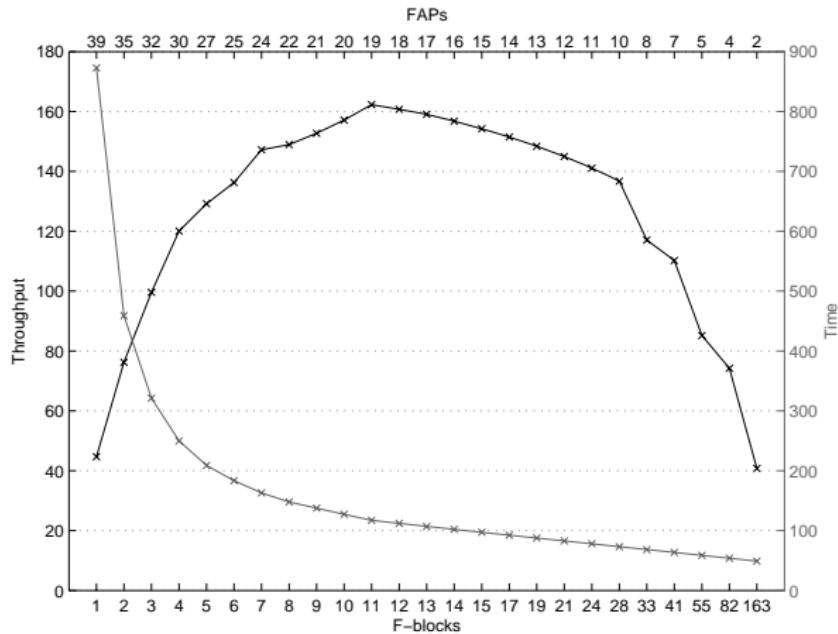


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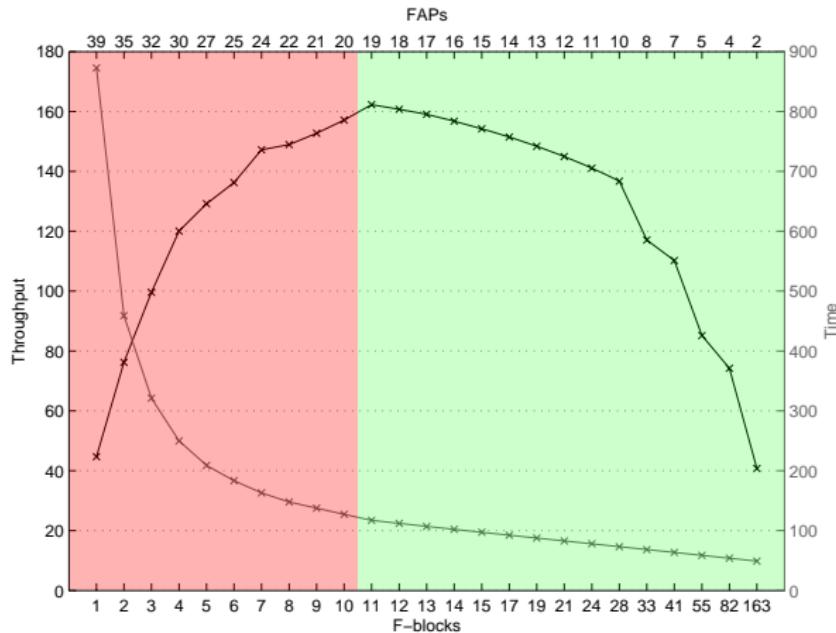
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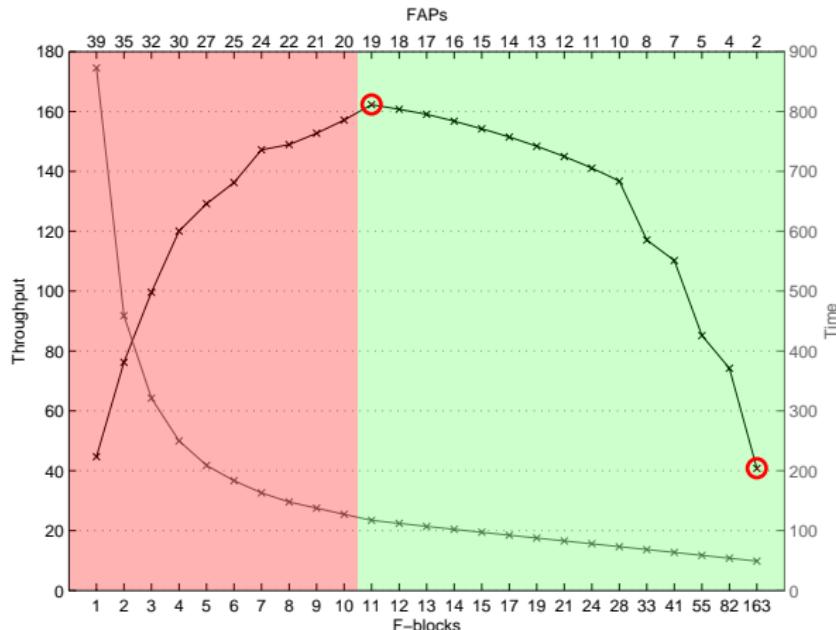
Parameters



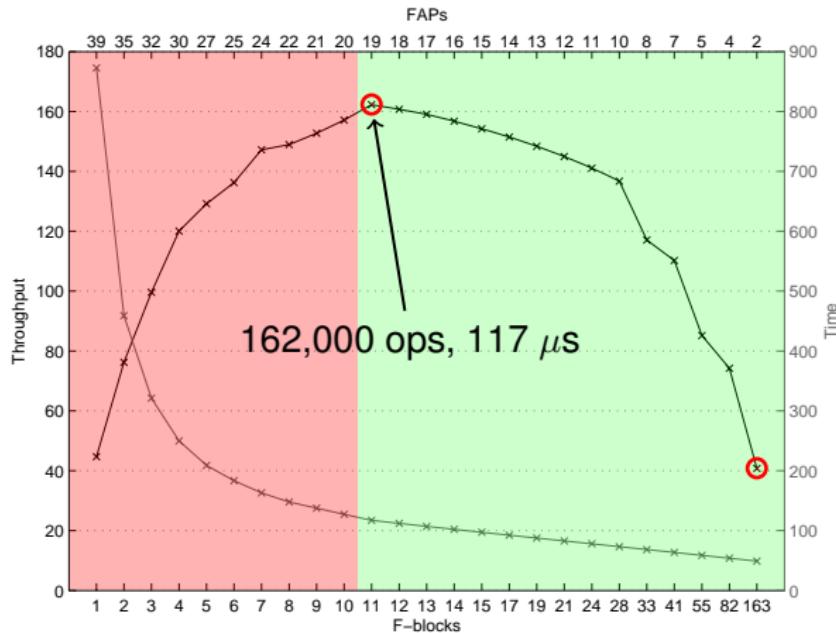
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Results on an Altera Stratix II S180C3

- VHDL
- Altera Quartus II 6.0 SP1

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Performance

- One verification 114.2 μ s (average)
- Up to 166,000 ops!

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- Very high ops achievable with modern FPGAs
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- Very high ops achievable with modern FPGAs
 - Development in FPGAs: speed and **area**
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- Future work:
 - Polynomial basis?
 - Counterpart implementations for signature generation
 - Other operations (hash, modular arithmetic)
 - Possible problems (side channel attacks, power, etc.)

Thank you.
Questions?