# Answer Set Programming

## *Implementation Techniques and Applications*

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- Introduction to Answer Set Programming (ASP)
- ASP with logic programs
- Implementation techniques
- Available systems
- Applications

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# Answer Set Programming

- Term coined by Vladimir Lifschitz
- Roots: KR, logic programming, nonmonotonic reasoning
- Based on some formal system with semantics that assigns a theory a collection of answer sets (models).
- An **ASP solver**: computes answer sets for a theory
- Solving a problem in ASP:
  Encode the problem as a theory such that **solutions** to the problem are given by **answer sets** of the theory.

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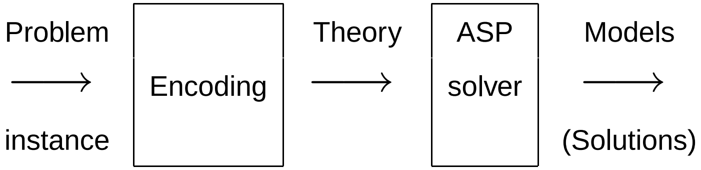
# ASP—cont'd

- Solving a problem using ASP



Problem $\longrightarrow$ instance | Encoding | Theory $\longrightarrow$ | ASP solver | Models $\longrightarrow$ (Solutions)

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| Possible formal system | Models |
|---|---|
| Propositional logic | Truth assignments |
| CSP | Variable assignments |
| Logic programs | Stable models |

# Example. $k$-coloring problem

- Given a graph $(V,E)$ find an assignment of one of $k$ colors to each vertex such that no two adjacent vertices share a color.
- Encoding 3-coloring using propositional logic

| For each vertex $v \in V$: | For each edge $(v,u) \in E$: |
|---|---|
| $v(1) \lor v(2) \lor v(3)$ | $\neg v(1) \lor \neg u(1)$ |
| $\neg v(1) \lor \neg v(2)$ | $\neg v(2) \lor \neg u(2)$ |
| $\neg v(1) \lor \neg v(3)$ | $\neg v(3) \lor \neg u(3)$ |
| $\neg v(2) \lor \neg v(3)$ | |

- 3-colorings of a graph $(V,E)$ and models of the encoding correspond:
  vertex $v$ colored with color $i$ iff $v(i)$ true in the model.

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# What is ASP Good for?

Search problems:

- Constraint satisfaction
- Planning, routing
- Computer-aided verification
- Security analysis
- Product configuration
- Combinatorics
- Diagnosis

☞ Declarative problem solving

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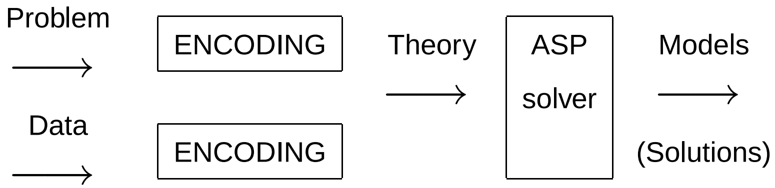
# Towards ASP in Practice

- Uniform encoding:
  separate problem specification and data
- Compact, easily maintainable representation
- Integrating KR, DB, and search techniques
- Handling dynamic, knowledge intensive applications:
  data, frame axioms, exceptions, defaults, closures



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# ASP Using Logic Programs

# ASP Using Logic Programs

- Logic programming: framework for merging KR, DB, and search
- PROLOG style logic programming systems not directly suitable for ASP:
  - search for proofs (not models) and produce answer substitutions
  - not entirely declarative
- In late 80s new semantical basis for “negation-as-failure” in LPs based on nonmonotonic logics: **Stable model semantics**
- Implementations of stable model semantics led to ASP

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# Example. $3$-coloring

**Problem**:

$$\begin{aligned} clrd(V,1) &\leftarrow \text{not } clrd(V,2), \text{not } clrd(V,3), vtx(V) \\ clrd(V,2) &\leftarrow \text{not } clrd(V,1), \text{not } clrd(V,3), vtx(V) \\ clrd(V,3) &\leftarrow \text{not } clrd(V,1), \text{not } clrd(V,2), vtx(V) \\ &\leftarrow edge(V,U), clrd(V,C), clrd(U,C) \end{aligned}$$

**Data**:

$$\begin{array}{lll} vtx(v) & vtx(u) & \ldots \\ edge(v,u) & edge(u,w) & \ldots \end{array}$$

☞ 3-colorings and stable models of the encoding correspond: $v$ colored $i$ iff $clrd(v,i)$ in the model.

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# LPs with Stable Models Semantics

- Consider normal logic program rules

$$A \leftarrow B_1, \ldots, B_m, \text{not } C_1, \ldots, \text{not } C_n$$

- Seen as constraints on an answer set (stable model):
  - if $B_1, \ldots, B_m$ are in the set and
  - none of $C_1, \ldots, C_n$ is included,

  then $A$ must be included in the set
- A stable model is a set of atoms
  (i) which satisfies the rules and
  (ii) where each atom is **justified** by the rules.

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# Stable Models — cont’d

- | Program: | Stable model: |
  |---|---|
  | $b \leftarrow$ | $\{b, f\}$ |
  | $f \leftarrow b, \text{not } eb$ | |
  | $eb \leftarrow p$ | |

- Another candidate model: $\{b, eb\}$ satisfies the rules but is not a proper stable model: $eb$ is included for no reason.
- Justifiability of stable models is captured by the notion of a **reduct** of a program
  ☞ The stable model semantics [Gelfond/Lifschitz,1988].

# Example. Stable models

- A program can have **none**, one, or **multiple** stable models.
- Program:

  $p_1 \leftarrow \text{not } q_1$

  $q_1 \leftarrow \text{not } p_1$

  Stable models:

  $\{p_1\}$

  $\{q_1\}$

- Program:

  $p_1 \leftarrow \text{not } q_1$

  $q_1 \leftarrow \text{not } p_1$

  $\leftarrow \text{not } p_1$

  $\leftarrow \text{not } q_1$

  Stable models:

  None

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# Variables

- Variables are needed for uniform encodings

  Program:

  $$
  \begin{aligned}
  &clrd(V,1) \leftarrow \text{not } clrd(V,2), \text{not } clrd(V,3), vtx(V)\\
  &clrd(V,2) \leftarrow \text{not } clrd(V,1), \text{not } clrd(V,3), vtx(V)\\
  &clrd(V,3) \leftarrow \text{not } clrd(V,1), \text{not } clrd(V,2), vtx(V)\\
  &\leftarrow edge(V,U), clrd(V,C), clrd(U,C)
  \end{aligned}
  $$

  Data:

  $$
  \begin{array}{lll}
  vtx(v) & vtx(u) & \ldots\\
  edge(v,u) & edge(u,w) & \ldots
  \end{array}
  $$

---

# Variables — cont’d

- Semantics: Herbrand models
- A rule is seen as a shorthand for the set of its ground instantiations.

**Example.**

$$clrd(V,1) \leftarrow \text{not } clrd(V,2), \text{not } clrd(V,3), vtx(V)$$

is a shorthand for

$$
\begin{aligned}
&clrd(v,1) \leftarrow \text{not } clrd(v,2), \text{not } clrd(v,3), vtx(v)\\
&clrd(u,1) \leftarrow \text{not } clrd(u,2), \text{not } clrd(u,3), vtx(u)\\
&clrd(1,1) \leftarrow \text{not } clrd(1,2), \text{not } clrd(1,3), vtx(1)\\
&\ldots
\end{aligned}
$$

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# Stable Models — cont’d

- A stratified program has a unique stable model (canonical model).
- It is **linear time to check** whether a set of atoms is a stable model of a ground program.
- It is **NP-complete to decide** whether a ground program has a stable model.
- Normal programs (without function symbols) give a **uniform solution** to every NP search problem.

# Extensions to Normal Programs

- **Classical negation**
  Can be handled by normal programs (renaming):

  $p \leftarrow \text{not } \neg p$ corresponds to $p \leftarrow \text{not } p'$

  $\leftarrow p, p'$

- **Encoding of choices**
  - Choice rules: $\{a\} \leftarrow b, \text{not } c$
  - Disjunctive rules: $a_1 \vee a_2 \leftarrow b, \text{not } c$
    - Higher expressivity and complexity ($\Sigma_2^p$)
    - Special purpose implementations (`dlv`)
    - Can be implemented also using an ASP solver for normal programs as the **core engine** (`GnT`)

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# Extensions — cont'd

- Many extensions implemented using an ASP solver as the **core engine**:
  - preferences
  - nested logic programs
  - circumscription, planning, diagnosis, . . .
- Aggregates
  - `count`
    Example: choose 2–4 hard disks
  - `sum`
    Example: the total capacity of the chosen hard disks must be at least 20 GB.
  - Built-in support for aggregates in the search procedures (`Smodels`, `dlv`)

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# Extensions — cont'd

- Optimization
  Example: prefer the cheapest set of hard disks
  (Built-in support in `Smodels`)
- Weak constraints with weight and priority levels

$$:\sim B_1, \ldots, B_m, \text{not } C_1, \ldots, \text{not } C_n [w : l]$$

(Built-in support in `dlv`)

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# Example. Rules in `Smodels`

- Cardinality constraints
  `2 {hd_1,...,hd_n } 4`
- Weight constraints
  `20 [hd_1 =6,...,hd_n = 13]`

  A.k.a. **pseudo-Boolean constraints**:

$$6hd_1 + \cdots + 13hd_n \geq 20$$

- Optimization
  `minimize [hd_1 = 100,...,hd_n = 600]`

# Generate-and-test programming

- Basic methodology:
  - **Generator rules**: provide candidate answer sets (typically encoded using choice constructs)
  - **Tester rules**: eliminate non-valid candidates (typically encoded using integrity constraints)
  - **Optimization statements**: Criteria for preferred answer sets (typically encoded using cost functions)

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# Example. $k$-coloring problem

- $k$-coloring: an assignment of one of $k$ colors to each vertex such that no two adjacent vertices share a color.
- Input: available colors and a graph
  - `color(1).,...,color(k).`
  - `vtx(v).,...`
  - `edge(v,u).,...`

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# $k$-coloring — cont'd

- An assignment of colors is represented by ground atoms of the form `clrd(v,c)` where `v` is a vertex and `c` is an available color.
- The basic idea of the encoding:
  (i) generator rules produce candidate stable models (assignments)
  (ii) tester rules eliminate candidates which do not satisfy the coloring condition.

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# $k$-coloring — cont'd

```
% Encoding of the k-coloring problem
% Generator: producing candidate stable models
1 {clrd(V,C):color(C)} 1 :- vtx(V).

% Tester: eliminate candidates
% not satisfying the coloring condition.
:- edge(V,U), color(C), clrd(V,C), clrd(U,C).
```

- Given the encoding program (the input facts and the generator and tester rules):
  **$k$-colorings and stable models correspond**.
- $k$-coloring: facts `clrd(v,c)` in the stable model.

# Example: Review assignment

```
% DATA:
reviewer(r1). ...
paper(p1). ...
classA(r1,p1). ... % Preferred papers
classB(r1,p2). ... % Doable papers
coi(r1,p3). ...    % Conflicts of interest

% PROBLEM
% Each paper is assigned 3 reviewers
3 { assigned(P,R):reviewer(R) } 3 :- paper(P).
% No paper assigned to a reviewer with coi
:- assigned(P,R), coi(R,P).
```

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# Review Assignment — cont’d

```
% No reviewer has an unwanted paper.
:- paper(P), reviewer(R),
   assigned(P,R), not classA(R,P), not classB(R,P).
% No reviewer has more than 8 papers
:- 9 { assigned(P,R): paper(P) }, reviewer(R).
% Each reviewer has at least 7 papers
:- { assigned(P,R): paper(P) } 6, reviewer(R).
% No reviewer has more than 2 classB papers
:-  3 { assignedB(P1,R): paper(P1) }, reviewer(R).
assignedB(P,R) :- classB(R,P), assigned(P,R).
% Minimize the number of classB papers
minimize [ assignedB(P,R):paper(P):reviewer(R) ].
```

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# ASP vs Other Approaches

- SAT, CSP, (M)IP
  - Similarities: search for models (assignments to variables) satisfying a set of constraints
  - Differences: no logical variables, database, DDB or KR techniques available, search space given by variable domains
- LP, CLP:
  - Similarities: database and DDB techniques
  - Differences: Search for proofs (not models), non-declarative features

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# Implementing ASP Solvers

# ASP Solvers

- ASP solvers need to handle two challenging tasks
  - complex data
  - search
- The approach has been to use
  - **logic programming and deductive data base techniques** for the former
  - **SAT/CSP related search techniques** for the latter
- In the current systems: separation of concerns

☞ A two level architecture

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# Architecture of ASP Solvers

Typically a two level architecture employed

- **Grounding** step handles complex data:
  - Given program $P$ with variables, generate a set of ground instances of the rules which preserves the models.
  - LP and DDB techniques employed
- **Model search** for ground programs:
  - Special-purpose search procedures
  - Translation to SAT

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# Model Search

Two promising approaches to model computing for ground programs

- Special purpose search procedures exploiting the particular properties of stable model semantics
- Translating the stable model finding problem to a propositional satisfiability problem exploiting state of the art SAT solvers

☞ These approaches are **closely related** via (Clark's) program **completion**

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# Program Completion

- Program completion $\mathrm{comp}(P)$: a simple translation of a logic program $P$ to a propositional formula.
  **Example.**

| $P:$ | $\mathrm{comp}(P):$ |
|---|---|
| $a \leftarrow b, \mathrm{not}\ c$ | $a \leftrightarrow ((b \land \neg c) \lor (\neg b \land d))$ |
| $a \leftarrow \mathrm{not}\ b, d$ | $\neg b, \neg c, \neg d$ |
| $\leftarrow a, \mathrm{not}\ d$ | $\neg(a \land \neg d)$ |

- **Supported models** of a logic program and **propositional models** of its completion coincide.
- For **tight programs** (no positive recursion) **supported and stable models** coincide (Fages).

# Program Completion — cont'd

- Stable models for tight programs can be computed using a SAT solver:
  - Form the completion and transform that to CNF (typically with new atoms).
  - Run a SAT solver on the CNF and translate results back.
- For tight programs: DPLL (`CMODELS`) on the translated CNF and ASP solver (`smodels`) on the original program are (propagation) **equivalent** [Giunchiglia and Maratea, ICLP05]

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# Program Completion — cont'd

- For non-tight programs (with positive recursion) **ASP** solvers have **more powerful propagation** techniques.
  **Example.**

|  |  |  |
|---|---|---|
| $p \leftarrow q$ |  | $p \leftrightarrow q$ |
| $q \leftarrow p$ | vs | $q \leftrightarrow p$ |
| ASP solver: |  | SAT solver: |
| unique model: $\{\}$ |  | 2 models: $\{\}, \{p, q\}$ |

- Positive recursion needed, e.g., for capturing **closures**: **reachability, transitive closure**

```
tc(X,Y) :- p(X,Y).
tc(X,Z) :- p(X,Y), tc(Y,Z).
```

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# Translations to SAT

- Translating non-tight LPs to SAT is challenging
  - Modular translations not possible (Niemelä, 1999)
  - Without new atoms exponential blow-up (Lifschitz and Razborov)
  - One-to-one correspondence between propositional models and answer sets non-trivial
- Approaches
  - Extend completion with **loop formulas dynamically** (`ASSAT`, `CMODELS`)
  - One pass compilation to SAT $O(\|P\| \times \log|At(P)|)$ translation (Janhunen, ECAI 2004)

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# SAT and ASP

Due to close relationship results carry over

- **Restarting** has been found useful in SAT/CSP
  **New version 2.31**: `smodels -restart`
- Modern SAT solvers employ **conflict driven learning and backjumping**
  First ASP attempt (Ward, Schlipf, 2004)
- SAT solvers use **watched literal** data structures to achieve efficient propagation for large clause sets
- ASP solvers have **built-in support for aggregates** (cardinality and weight constraints)
  Efficient techniques for pseudo-Boolean constraints

# `smodels` System

(`http://www.tcs.hut.fi/Software/smodels`)

program (variables) → `lparse` front-end → ground program → `smodels` search → stable models

- Front-end: (deductive) DB techniques for stratified programs
- Special purpose search engine:
  - array data structures (Dowling-Gallier type)
  - local computations for large rule sets
  - linear space requirements
  - optimization built-in

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# `smodels` System—cont'd

- `smodels`
  - latest version 2.31
  - `-restart` option
  - `-nolookahead` optio
    lazy lookahead heuristics
    (approximates full lookahead)
- `lparse`
  - latest version 1.0.17
  - domain-restricted programs
  - function symbols and conditional literals
  - built-in predicates/functions (comparisons, arithmetic)

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# Other ASP Implementations

| | |
|---|---|
| `dlv` | `http://www.dbai.tuwien.ac.at/proj/dlv/` |
| `GnT` | `http://www.tcs.hut.fi/Software/gnt/` |
| `CMODELS` | `http://www.cs.utexas.edu/users/tag/cmodels.html` |
| `ASSAT` | `http://assat.cs.ust.hk/` |
| `nomore++` | `http://www.cs.uni-potsdam.de/nomore/` |
| `XASP` | distributed with XSB v2.6<br>`http://xsb.sourceforge.net` |
| `aspps` | `http://www.cs.engr.uky.edu/ai/aspps/` |
| `pbmodels` | `http://www.cs.engr.uky.edu/ai/pbmodels/` |
| `ccalc` | `http://www.cs.utexas.edu/users/tag/cc/` |

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# Applications

# Applications

- Planning
  USAdvisor project at Texas Tech:
  A decision support system for the flight controllers of space shuttles
- Product configuration
  –Intelligent software configurator for Debian/Linux
  –WeCoTin project (Web Configuration Technology)
  –Spin-off (`http://www.variantum.com/`)
- Computer-aided verification
  –Partial order methods
  –Bounded model checking

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# Applications—cont’d

- VLSI routing
- Planning
- Combinatorial problems, network management, network security, security protocol analysis, linguistics . . .
- C. Baral. Knowledge Representation, Reasoning and Declarative Problem Solving. Cambridge University Press, 2003.
- Applying ASP
  - as a stand alone system
  - as an embedded solver

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# Conclusions

**ASP = KR + DB + search**

- ASP emerging as a viable KR tool
- Efficient implementations under development (`Smodels`, `aspps`, `dlv`, `XASP`, `CMODELS`, `ASSAT`, `nomore++`, `...`)
- Expanding functionality and ease of use
- Growing range of applications

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# Topics for Further Research

- Intelligent grounding
- Model computation without full grounding
- Program transformations, optimizations
- Model search: learning, restarting, backjumping, heuristics, local search techniques
- Distributed and parallel implementation techniques
- Language extensions
- Programming methodology
- Tool support