Prioritized Default Theories : a new characterization.

F. Benhammadi
LERIA, Université d'Angers
2, bd Lavoisier, F-49045
Angers cedex 01
farid.benhammadi@univ-angers.fr

P. Nicolas
LERIA, Université d'Angers
2, bd Lavoisier, F-49045
Angers cedex 01
pascal.nicolas@univ-angers.fr

T. Schaub
Institut für Informatik,
Universität Potsdam
Postfach 60 15 53, D-14415 Potsdam
torsten@cs.uni-potsdam.de

Abstract

Default logic is one of the best known and used formalisms for default reasoning. The interest of this paper is to examine a new method for reasoning in default logic in presence of priorities which are represented by an arbitrary partial order on default rules. We propose here an alternative approach for computing extensions from the prioritized default theories initially proposed by Brewka [Brewka,1994]. By this way, we obtain two results, one for extension calculus, and another for query answering. Given a prioritized default theory, in the first case we are able to build more efficiently all its extensions, and in the second case we are able to compute the prioritized default proof of a given formula.

Keywords Knowledge Representation, Logic for artificial Intelligence, non-monotonic Reasoning, Default logic, Priorities

1 Introduction

Reiter's default logic is one of the best known and most-widely used formalisms for default reasoning. Default logic is very expressive and so, for instance, provides semantics for extended logic programming [Gelfond & Lifschitz,1990], inheritance reasoning [Etherington & Reiter,1983] and other logical formalisms involving reasoning by default.

In default logic, knowledge is represented by means of a set of closed formulas W and a set of *default rules* of the form $\frac{\alpha:\beta}{\gamma}$, where α, β, γ are formulas respectively called the *prerequisite*, the *justification* and the *consequent*. The defaults in D induce one (or multiple) *extensions* of the facts in W. Any such extension, E say, is a deductively closed set of formulas containing W such that for any $\frac{\alpha:\beta}{\gamma} \in D$, if $\alpha \in E$ and $\neg\beta \notin E$ then $\gamma \in E$.

Despite default logic's popularity, we encounter severe deficiencies in the most general setting. For instance, default logic does not guarantee the existence of extensions. Moreover, it does not address the notion of *specificity*, which is a fundamental principle in common sense reasoning according to which more specific defaults should be preferred over less specific ones.

For avoiding the first problem, it is usually sufficient to restrict oneself to a subclass of default logic given by so-called *normal default theories*. Such theories consist of defaults of the form $\frac{\alpha:\beta}{\beta}$. This class enjoys several desirable properties not present in the general case. Apart from guaranteeing the existence of extensions, normal default theories are semi-monotonic (ie. monotonic wrt the addition of defaults) and the resulting extensions are always mutually contradictory to each other. In particular the former property is very desirable from a computational point of view, since it allows us to restrict our attention to default rules relevant for proving a query (cf. [Reiter,1980]). In general, this restriction makes sense whenever there is no need for expressing priorities between defaults, since this is usually the reason for non-normal defaults (cf. [Reiter & Criscuolo,1981; Brewka,1991]).

Unfortunately, the restriction to normal default theories (or any other subclass) does not work for addressing the failure of specificity. As an example, consider the statements "students are typically not employed", "students are typically adults" and "adults are typically employed" along with the corresponding default theory:

$$\begin{aligned} W &= \{S\} \\ \Delta &= \left\{\tfrac{S:\neg E}{\neg E}, \tfrac{S:A}{A}, \tfrac{A:E}{E}\right\} \end{aligned} \qquad (1)$$

Intuitively, we would like to derive A and $\neg E$. We do not want to derive E, since this violates the principle of specificity. In fact, being a student is more specific than being an adult and so we want to give the first default priority over the third one. However, this theory gives two extensions, $Th(\{A, \neg E, S\})$ and $Th(\{A, E, S\})$, showing that also normal default theories cannot deal with specificity.

Recently, a couple of approaches were developed for isolating specificity information (cf. [Brewka,1994; Delgrande & Schaub,1994]) out of a given set of defaults. In the above example these methods allow us to detect automatically that the default $\frac{S:\neg E}{\neg E}$ is more specific than the default $\frac{A:E}{E}$. This corresponds to the intuition sketched above. In [Brewka,1994], this specificity information is expressed by means of a strict partial order $<$. Then, $\frac{S:\neg E}{\neg E} < \frac{A:E}{E}$ expresses the fact that the first default is more specific than the second one. In this way, one obtains so-called *prioritized normal default theories*, ie. normal default theories along with a strict partial order on the defaults. For describing prioritized extensions, Brewka introduces in [Brewka,1994], a characterization of extensions respecting the specificity information given by such a partial order. As shown in Section 2, this approach eliminates the second extension of default theory (1) and only gives the first, more specific extension.

In order to allow for query-answering from prioritized default theories, [Schaub,1994] has modified and extended the approach of [Rothschild & Schaub,1993]. This is achieved by identifying the characteristic features of Brewka's approach to integrating specificity into default logic.

The paper is organized as follows. In Section 2, we review the approach given in [Brewka,1994]. Section 3 provides an alternative characterization of extensions from prioritized default theories. It gives an algorithm to compute extensions of prioritized default theories and a method to treat the query answering problem in presence of prioritized defaults.

2 Prioritized default theories

The approach of [Brewka,1994] combines and extends the work described in [Brewka,1989] and [Brewka,1991] for handling priorities in default reasoning. In particular, the latter approaches were mainly aiming at prerequisite-free normal defaults.[1] An alternative approach to prioritized default theories is proposed in [Baader & Hollunder,1993]. In fact, it is shown in [Baader & Hollunder,1993] that both approaches are orthogonal to each other even though they share many common intuitions.

According to [Brewka,1994], the basic idea is as follows: *"during the generation of an extension defaults with higher priorities whose prerequisite has been derived are always considered before defaults of lower priority. This guarantees that defaults of lower priority are applied only if this does not lead to a conflict with a 'better' default. Since the partial ordering usually leaves some priorities undecided we have to take every total ordering of the defaults into account that respects the original partial order. Every such total order will give rise to one extension. Different orderings may generate the same extension."*

As defined in [Brewka,1994], a prioritized default theory is a triple $(\Delta, W, <)$, where Δ is a finite set of normal defaults[2], W a set of formulas, and $<$ a strict partial order on Δ. As mentioned in the introduction, a more specific default δ gets priority over a less specific default δ'; this is expressed by $\delta < \delta'$.

Now, a central concept is that of *activeness*:

Definition 2.1 *A normal default rule δ is* active *in a set of formulas E iff $Prereq(\delta) \in E$, $\neg Conseq(\delta) \notin E$, and $Conseq(\delta) \notin E$.*

By means of this notion, and for any total extension $\ll$ of $<$, [Brewka,1994] defines a Brewka's prioritized extension (denoted in the following by B-extension) of $(\Delta, W, <)$ in the following way.

Definition 2.2 *Let $(\Delta, W, <)$ be a prioritized default theory and let $\ll$ be a strict total order containing $<$. Define*

$E_0 = Th(W)$

and for $i \geq 0$

$$E_{i+1} = \begin{cases} E_i \text{ if there is no } \delta \in \Delta \text{ active in } E_i \\ \\ Th(E_i \cup \{Conseq(\delta)\}) \text{ otherwise,} \\ \text{where } \delta \text{ is the } \ll\text{-minimal default} \\ \text{rule that is active in } E_i. \end{cases}$$

Then, E is a prioritized extension of $(\Delta, W, <)$ iff

$E = \bigcup_{i=0}^{\infty} E_i$

The sequence of defaults $\langle \delta_0, \ldots, \delta_n \rangle$ inducing the se-

[1] According to [Brewka,1994], the approach given in [Brewka,1991, Section 3] leads to reasonable results for prerequisite-free normal defaults only, although it applies to normal defaults, too.

[2] As argued in [Brewka,1994], priorities compensate the additional expressiveness of non-normal defaults so that he confines himself to normal ones.

quence $\langle E_1, \ldots, E_{n+1}\rangle$ is called a set of *ordered generating defaults* $OGD(\Delta, E, \ll)$ when n is the smallest integer such that $\forall k > n\ E_k = E_n$.

As shown in [Brewka,1994], this definition yields the same extensions as in regular default logic in the case of an empty order $<$.[3] Hence, prioritized default logic is a true extension of regular default logic. Also, any prioritized extension of a theory $(\Delta, W, <)$ is a regular extension of (Δ, W). Moreover, the existence of prioritized extensions is guaranteed and prioritized default logic is semi-monotonic wrt the order.

Consider our initial example in (1), the default theory $\left(\left\{\frac{S:\neg E}{\neg E}, \frac{S:A}{A}, \frac{A:E}{E}\right\}, \{S\}\right)$. This theory has two extensions in regular default logic, namely $Th(\{A, \neg E, S\})$ and $Th(\{A, E, S\})$. Now using the formalisms developed in [Brewka,1994; Delgrande & Schaub,1994], we fix a precedence between the first and the third default, ie. $\frac{S:\neg E}{\neg E} < \frac{A:E}{E}$. This yields the only following prioritized default theory:

$$\begin{array}{rcl} W & = & \{S\} \\ \Delta & = & \left\{\frac{S:\neg E}{\neg E}, \frac{S:A}{A}, \frac{A:E}{E}\right\} \\ < & = & \left\{\left(\frac{S:\neg E}{\neg E} < \frac{A:E}{E}\right)\right\} \end{array} \tag{2}$$

For brevity, we denote the defaults in (2) by $\delta_1, \delta_2, \delta_3$ from left to right. Then, from the order given above, we obtain three total orders respecting $<$, namely

$$\begin{array}{l} \delta_2 \ll \delta_1 \ll \delta_3, \\ \delta_1 \ll \delta_2 \ll \delta_3, \\ \delta_1 \ll \delta_3 \ll \delta_2. \end{array}$$

It is easy to verify that for each total order, we obtain the first and more specific extension, $Th(\{A, \neg E, S\})$, as the only prioritized extension. As an example, the first of the three total orders above yields the following sequence of partial prioritized extensions:

$$\begin{array}{l} E_0 = Th(\{S\}), \\ E_1 = Th(\{S, A\}), \\ E_2 = Th(\{S, A, \neg E\}), \\ E_3 = E_2, \\ \ldots \end{array}$$

The two other orders lead to the same extension. Thus in all three cases, the default $\frac{S:\neg E}{\neg E}$ takes priority over the less specific default $\frac{A:E}{E}$.

[3]Here and in what follows, we restrict our attention to normal default theories in regular default logic.

In general, the two features distinguishing default rules from classical implications are the *character of an inference rule* and the additional *consistency check.* While the latter is handled in default logic in the usual manner by testing satisfiability, the former needs a more subtle treatment: The character of an inference rule can be captured by the notion of *groundedness* defined by [Schwind,1990]. We call a set of default rules Δ *grounded* in a set of facts W iff there exists an enumeration $\langle\delta_i\rangle_{i\in I}$ of Δ such that

$$W \cup Conseq(\{\delta_0, \ldots, \delta_{i-1}\}) \vdash Prereq(\delta_i) \tag{3}$$

for $i \in I$. For convenience, we denote the prerequisite of a default rule δ by $Prereq(\delta)$ and its consequent by $Conseq(\delta)$.[4] Groundedness distinguishes default rules from classical implications. For instance, the above default rule $\frac{A:E}{E}$ is (trivially) not grounded in the set of facts $\{\neg E\}$ so that reasoning by contraposition becomes impossible. That is, $\neg A$ is not derivable from $\neg E$.

So, in regular default logic, *groundedness* and *consistency* constitute the two characteristics, or better, the two qualifying conditions for the application and the use of default rules. In particular, these two notions allow for characterizing extensions in a very elegant way. As given in [Schwind & Risch,1994], an extension is characterized as $Th(W \cup Conseq(\Delta'))$, the deductive closure of the facts W and the consequents of a maximal set of default rules Δ' which is grounded and preserves consistency.

What is the additional qualifying condition needed for incorporating priorities? Here, the idea is to isolate sequences of default rules that respect the given partial order. In contrast to the independence of the concepts of groundedness and consistency, however, we encounter a more difficult situation if we want to preserve certain priorities. This is so because a priority between defaults is only effective if both defaults are applicable, ie. if their prerequisites are derivable and their consequents are consistent. In this way, the preservation of priorities becomes dependent on the concepts of groundedness and consistency. Let's note that [Schaub,1994] gives a characterization of prioritized extensions that turns out to approximate the ones of [Brewka,1994]. This is due to the lack of an incremental accumulation of the constraints on the initial partial order.

[4]Recall that we deal with normal defaults for which the consequent is equivalent to the justification.

3 An alternative characterization of prioritized extensions

In what follows, we define an alternative approach for computing extensions from prioritized normal default theories and we use this result to adapt the query answering approach to prioritized default theories.

3.1 Theoretical framework.

In order to avoid the total order$\ll$ for the prioritized extension characterization, we exclusively use in our approach a strict partial order $<$ which is augmented during the generation of these extensions. By using the concept of activeness in definition 2.1, we define the set of minimal active defaults as follows:

Definition 3.1 *Let $(\Delta, W, <)$ be a prioritized default theory and let S be a set of formulas. The set of minimal active defaults is defined as follows:*

$$\mathsf{MAD}(\Delta, S, <) = \left\{ \delta \in \Delta \;\middle|\; \begin{array}{l} \delta \textit{is active in } S \textit{ and for} \\ \textit{each } \delta' \in \Delta \textit{ such that} \\ \delta' < \delta, \delta' \textit{ is not active} \\ \textit{in } S. \end{array} \right\}$$

And now, we can give our alternative characterization of an extension in normal default logic with explicit priorities.

Theorem 3.1 *Let $(\Delta, W, <)$ be a prioritized normal default theory and let E be a set of formulas. Define*

$E_0 = Th(W)$, $<_0 = <$,

and for $i \geq 0$

if $\mathsf{MAD}(\Delta, E_i, <_i) = \emptyset$ then

$$\begin{cases} E_{i+1} = E_i \\ <_{i+1} = <_i \end{cases}$$

else for an arbitrary default $\delta \in \mathsf{MAD}(\Delta, E_i, <_i)$

$$\begin{cases} E_{i+1} = Th(E_i \cup \{Conseq(\delta)\}) \\ <_{i+1} = (<_i \cup \{(\delta, \delta_j) \mid \delta_j \in \mathsf{MAD}(\Delta, E_i, <_i) \setminus \{\delta\}\})^* \end{cases}$$

(represents the transitive closure)*

Then, E is the B-extension of $(\Delta, W, <)$ iff

$E = \bigcup_{i=0}^{\infty} E_i$.

Before to establish a formal proof of this result we can say that the most important point is that we do not use a total order $\ll$, but we increase step by step the initial partial order $<$. At each step we choose one of the active defaults that is minimal for the current priorities between the defaults. And then, after having applied this default, we record that it has become more prior than all other minimal active defaults by modifying the order $<_i$ in $<_{i+1}$. So, starting from $<$, we end with a prioritized extension E and a partial order $<_n$ that is a subset of a total order $\ll$. And, this total order gives the same extension E in the Brewka's method.

Proof of theorem 3.1

$\Longrightarrow$ Let $E = \bigcup_{i=0}^{\infty} E_i$ a B-extension of $(\Delta, W, <)$ based on a total order $\ll$ containing $<$ and for which the sequence of generating defaults $DG(\Delta, E, \ll) = \langle \delta_0, \ldots, \delta_n \rangle$. E_i represents a partial extension in Brewka's definition and E'_i represents a partial extension built by our definition given in theorem 3.1. By induction, we show that $E_i = E'_i$ and $<_i \subseteq \ll$ for $i \geq 0$. We want to show that there is one way for choosing δ in our approach such that it builds the sequence $\langle E'_0, \ldots, E'_{n+1} \rangle$ and $\langle \delta'_0, \ldots, \delta'_n \rangle$ with $E = \bigcup_{i=0}^{\infty} E'_i$ and $\delta'_i = \delta_i$ for $i = 0, \ldots, n$ and the final order $<_n$ is a sub-order of $\ll$.

Base: By definition, we have $E_0 = Th(W) = E'_0$ and also $<_0 = <$, therefore $<_0 \subseteq \ll$.

Hypothesis: Let us assume $E_i = E'_i$, $\delta_i = \delta'_i$ and the order $<_i \subseteq \ll$ for $i = 0, \ldots, n$. We distinguish the two following cases:

1. No default is active in E_i, thus one has $E_{i+1} = E_i$ and there is no new default in the sequence $\langle \delta_0, \ldots, \delta_n \rangle$.. Furthermore, no default is active in E'_i as by hypothesis $E_i = E'_i$, then, $\mathsf{MAD}(\Delta, E'_i, <_i) = \emptyset$ and $E'_{i+1} = E'_i$. So, obviously $E_{i+1} = E'_{i+1}$. One has $<_{i+1} \subseteq <_i$ and $\mathsf{MAD}(\Delta, E'_i, <_i) = \emptyset$ and the hypothesis $<_i \subseteq \ll$ implies that $<_{i+1} \subseteq \ll$.

2. Let δ be the $\ll$-minimal default that is active in E_i. By hypothesis, we have $\mathsf{MAD}(\Delta_i, E'_i, <_i) = \mathsf{MAD}(\Delta_i, E_i, <_i)$, therefore, $\delta \in \mathsf{MAD}(\Delta_i, E'_i, <_i)$ because if this is not the case there is an active default $\delta^* \in \Delta$ such that $\delta^* < \delta$ and δ will not be $\ll$-minimal default active in E_i(contradiction). So, as $\delta \in \mathsf{MAD}(\Delta_i, E'_i, <_i)$, one chooses it by taking $\delta'_i = \delta$ with $E'_{i+1} = Th(E'_i \cup \{Conseq(\delta)\})$ and therefore $E_i = E'_i$. One obtains $E_{i+1} = E'_{i+1}$. Consider $<_{i+1} = <_i \cup \{(\delta, \delta_j) \mid \delta_j \in \mathsf{MAD}(\Delta, E_i, <_i) \setminus \{\delta\}\}$. By hypothesis, $<_i$ is a sub-order of $\ll$. If $<_{i+1}$ is not a subset of $\ll$ then there exists one of relation $\delta < \delta_j$ such that $\delta_j \ll \delta$. But since δ_j is active, this contradicts the fact that δ is a $\ll$-minimal default that is active in E_i, so $<_{i+1} \subseteq \ll$.

$\Longleftarrow$ Let $E' = \bigcup_{i=0}^{\infty} E'_i$ be an extension of $(\Delta, W, <)$ and a partial order $<_n$ obtained by using theorem 3.1. Then, we have to show that there exists a Brewka's extension $E = E'$ with $DG(\Delta, E, \ll) = \langle \delta'_0, \ldots, \delta'_n \rangle$. Let $\ll$ be a total order containing a partial order $<_i$ and let $\mathcal{B}$ be the set of total orders containing the partial order $<$ and let $\mathcal{T}$ be the set of total orders containing the partial order $<_n$. It is obvious that since $< \subseteq <_n$ one has $\mathcal{T} \subseteq \mathcal{B}$. Therefore $\ll$ is a total order containing $<$.

Now, we have to show by induction the equality between the B-extension E generated by the total order $\ll$ and E'.

Base: $E'_0 = Th(W)$ and $E_0 = Th(W)$, thus $E'_0 = E_0$.

Hypothesis: Let we assume that $E'_i = E_i$.

1. If $\mathit{MAD}(\Delta_i, E'_i, <_i) = \emptyset$ we have $E'_{i+1} = E'_i$. Therefore, there is no active default in E'_i and E_i, so $E'_{i+1} = E_{i+1}$.

2. If $\mathit{MAD}(\Delta_i, E'_i, <_i) \neq \emptyset$ we choose an arbitrary δ^* in $\mathit{MAD}(\Delta_i, E'_i, <_i)$ and we will get $E'_{i+1} = Th(E'_i \cup \{Conseq(\delta^*)\})$. As one adds the relations $\delta^* < \delta_j$ for all $\delta_j \in \mathit{MAD}(\Delta_i, E'_i, <_i)$, then these relations will be in the total order $\ll$. Formally, $\delta^* \ll \delta_j$ since $<_i \subseteq <_n$ where $\ll$ is a total order containing $<_n$. Thus, δ^* is surely a $\ll$ -minimal default that is active in E_i and such that

   (a) one has $E'_i = E_i$ by hypothesis
   (b) $E_{i+1} = Th(E_i \cup \{Conseq(\delta)\})$

   one obtains $E'_{i+1} = E_{i+1}$ $\square$

The Figure 1 illustrates our new characterization on student example in (2).

Starting from the facts $\{S\}$, we obtain two active default rules, δ_1 and δ_2. Since neither is preferred over the other one, both constitute $<$-minimal default rules. This is indicated by the set $\mathit{MAD}(\Delta, \{S\}, <_0)$. So that we can apply either of them. Choosing δ_2 renders it inactive and furthermore activates δ_3, since now its prerequisite is derivable from $\{S\} \cup \{A\}$ (ie. the facts along with the consequent of δ_2). $<_1$ is obtained by adding to $<_0$ the relation $\delta_2 < \delta_1$ and computing the transitive closure of this set. Since δ_1 is still active, we obtain two active defaults, δ_1 and δ_3. Now, δ_1 has to be applied since it is preferred over δ_3 by $\delta_1 < \delta_3$. Finally, no default is active anymore and we obtain the

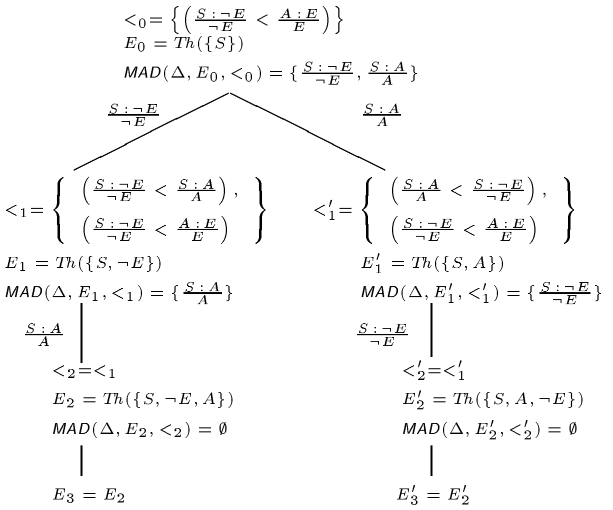

Figure 1: Computing extensions of the prioritized default theory 2

enumeration $\langle \delta_2, \delta_1 \rangle$ that induces a prioritized extension containing $\{S\} \cup \{A, \neg E\}$. The other branch of the tree gives the same extension and is obtained by choosing δ_1 previously than δ_2.

3.2 Extension calculus algorithm.

Now, we can present the following algorithm that computes all prioritized extensions from a prioritized normal default theories $(\Delta, W, <)$.

Procedure Computing_extensions$(\Delta, W, <)$
begin
 $E \leftarrow Th(W)$
 if $\mathit{MAD}(\Delta, E, <) \neq \emptyset$
 then
  for all $\delta \in \mathit{MAD}(\Delta, E, <)$
  begin
   $E' \leftarrow E \cup \{Conseq(\delta)\}$
   $<' \leftarrow (< \cup \{(\delta, \delta_j) \mid \delta_j \in \mathit{MAD}(\Delta, E, <) \setminus \{\delta\}\})^*$
   Computing_extensions$(\Delta \setminus \{\delta\}, E', <')$
  end
 else
   "Write(E)"
 endif
end

Why our approach is more efficient to compute all extensions than the original one ? In Brewka's original method it is possible that k different total orders issued from the given partial one produces the same extension E. In this case E is computed k different times. But, if these different orders are produced by some non ordered defaults that are never active, then our method does not examine them since it is only centered on active defaults in first. So, the same extension E will be computed only once.

This attractiveness is showing in the next example:

Example 3.1 *Let consider the following default theory:*

$$\begin{aligned} W &= \emptyset \\ \Delta &= \left\{\frac{:A}{A}, \frac{:\neg A}{\neg A}, \frac{C:B}{B}, \frac{\neg A:\neg B}{\neg B}, \frac{A:C}{C}\right\} \\ < &= \left\{\left(\frac{\neg A:\neg B}{\neg B} < \frac{C:\neg B}{\neg B}\right)\right\} \end{aligned} \quad (4)$$

From this partial order, we obtain sixty total orders that give only two different prioritized extensions: $Th(\{A, C, B\})$ and $Th(\{\neg A, \neg B\})$. But, one must do sixty different calculus if we want to be sure to have computed all extensions. In the other hand, in our approach, it is not necessary to verify the sixty total orders because we generate only two sequences of ordered generating defaults.

In the case of prerequisite-free normal prioritized default theories, our approach coincides with Brewka's approach, because the number of the applied orders and the number of total orders are equal.

3.3 Query-answering in prioritized default logic

Non monotonic reasoning is not always concerned by the calculus of a whole set of conclusions from some premises. Sometimes, we want to be able to answer *yes* or *no* to the query "is formula φ a theorem" ? For this problem in normal default logic, Reiter has introduced the notion of default proof [Reiter,1980]. Then, we give below a definition of such a default proof in presence of priorities.

Definition 3.2 *The sequence of defaults $\langle\delta_1, \ldots, \delta_n\rangle$ is a prioritized default proof of φ in $(\Delta, W, <)$ iff*

- $W \cup Conseq(\{\delta_1, \ldots, \delta_n\}) \vdash \varphi$
- *$\langle\delta_1, \ldots, \delta_n\rangle$ is grounded in W (see (3))*
- *there exists a total order $\ll$ issued from $<$ and inducing a prioritized extension E of $(\Delta, W, <)$ such that its ordered generating default set is $OGD(\Delta, E, \ll) = \langle\ldots, \delta_1, \ldots, \delta_n, \ldots\rangle$*

As in classical normal default logic, we define a prioritized default proof as the "*beginning*" of a sequence of defaults that generates an extension containing the formula φ. So, the prioritized default proof obviously satisfies the regularity condition : $W \cup Conseq(\{\delta_1, \ldots, \delta_n\})$ is consistent.

At this point, it is important to note that introducing priorities in the query answering approach in default logic is a very difficult task because the two following points are difficultly compatible and mergeable.

- Searching a default proof of φ is done by starting from φ and trying to reach W by using some necessary defaults.
- Managing priorities is done by starting from W and firing defaults respectively to the priorities between them.

In view of these remarks we have decided to search a prioritized default proof in two steps : first we compute a classical default proof P of φ, and after we check if P can be transformed in a prioritized default proof P'. This method is described in the following algorithm.

Procedure Check_Proof$(\Delta, i, n, P, <, P', Success)$
{The first call to Check_Proof is done with
$P = \langle\delta_1, \ldots, \delta_n\rangle$ a classical default proof containing n defaults,
$i = 0$, $<$ the initial partial order, $P' = \langle\rangle$, $Success = false$}
begin
if $i = n$
then
$Success \leftarrow true$ {*P' contains the prioritized default proof*}
else
$E \leftarrow Th(W \cup Conseq(P'))$
if $\delta_{i+1} \in MAD(\Delta, E, <)$
then
$Check\_Proof\left(\begin{array}{l}\Delta, i+1, n, P, \\ (< \cup\{(\delta_i, \delta_k) \mid \delta_k \in MAD(\Delta, E, <) \setminus \{\delta_i\}\})^*, \\ P' \cup \{\delta_{i+1}\}, Success\end{array}\right)$
{*We progress simultaneously in P and P'.*}
else
$D \leftarrow \left\{\begin{array}{l}\delta \in MAD(\Delta, E, <) s.t. \\ W \cup Conseq(P \cup P' \cup \{\delta\}) \text{is consistent}\end{array}\right\}$
{*We keep only minimal active defaults compatible with P.*}
while $D \neq \emptyset$ and not Success **do**
$\delta \leftarrow$ an arbitrary element of D
$D \leftarrow D \setminus \{\delta\}$
$Check\_Proof\left(\begin{array}{l}\Delta, i, n, P, \\ (< \cup\{(\delta, \delta_k) \mid \delta_k \in MAD(\Delta, E, <) \setminus \{\delta\}\})^*, \\ P' \cup \{\delta\}, Success\end{array}\right)$
{*We progress only in P'.*}
endwhile
endif
endif
end

This algorithm has been implemented in Prolog and uses our system XRay [5] to build the initial classical default proof of φ.

Example 3.2 *Let's consider the following default theory.*

$$\begin{array}{rcl} W & = & \{\neg d \vee \neg b\} \\ \Delta & = & \left\{\frac{:a}{a}, \frac{:c}{c}, \frac{c:d}{d}, \frac{a:b}{b}, \frac{:\neg d}{\neg d}\right\} \\ < & = & \left\{\left(\frac{c:d}{d} < \frac{:a}{a}\right), \left(\frac{:c}{c} < \frac{a:b}{b}\right)\right\} \end{array} \quad (5)$$

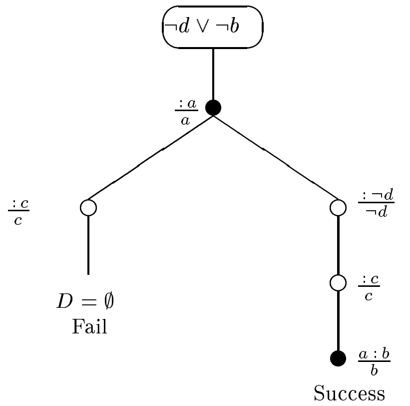

Figure 2: Transformation of a classical default proof in a prioritized one in example (5).

The figure 2 illustrates the major step of the verification of the validity of the classical proof $\langle \frac{:a}{a}, \frac{a:b}{b} \rangle$ of formula b in prioritized default theory (5). The resulting prioritized default proof is $\langle \frac{:a}{a}, \frac{:\neg d}{\neg d}, \frac{:c}{c}, \frac{a:b}{b} \rangle$ and contains the two defaults (black points) from the initial proof. In the first branch, using default $\frac{:c}{c}$ prevents to use the only active minimal default $\frac{c:d}{d}$ at the next step because $W \cup Conseq(\{\frac{:a}{a}, \frac{:c}{c}, \frac{a:b}{b}, \frac{c:d}{d}\})$ is inconsistent.

It is easy to see that our algorithm works in a same way as the algorithm of section (3.2) by trying to build the beginning of a prioritized extension. As previously, it is important to note that we start with the partial order between defaults, and we enlarge this order step by step during the building of the prioritzed proof. The search is driven by the given classical proof and that ensures the correctness of the method. Its completeness is submitted to the completeness of the classical proof search because when we fail to transform P into a prioritized proof P', we must generate another classical proof P and restart the checking step, etc...

[5] In [Schaub & Nicolas,1997] we have previously described XRay that is a tool to produce default proof in every semi-monotonic variants of default logic.

4 Conclusion

In order to avoid the recourse of the total order for computing the prioritized extensions from normal default theories in Brewka's approach, we have proposed a new approach which is essentially based on a strict partial order. Thus, the need for computing a total extension $\ll$ of an order $<$ for computing prioritized extension is dropped. Our new characterization allows us to produce an algorithm to compute all extensions and an algorithm to build a default proof of a formula in a context of a prioritized default theory.

Further works are envisaged to implement this treatment of priorities in our query answering system XRay and to try to avoid the bottom-up step of proof checking in order to have a completely local mechanism of default proof with priorities.

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