T-79.5204 **Spring 2007**
Combinatorial Models and Stochastic Algorithms
Tutorial 6, March 1
Problems

1. Consider the ensemble of N-bit binary strings $\Omega = \{0,1\}^N$ with Hamiltonian $H(\omega) =$ number of 1's in string ω. Compute explicitly the partition function Z_β for this ensemble, and derive expressions for its macroscopic total energy U_β and entropy S_β assuming the Gibbs distribution on microstates. Solve the equation $U_\beta = h$ for β.

2. Show that in any ensemble of microstates with the Gibbs probability distribution, one obtains

$$\langle H^2 \rangle = \frac{1}{Z} \frac{\partial^2 Z}{\partial \beta^2}$$

and, using this result, that the *specific heat* of the system,

$$C_V = \frac{\partial \langle H \rangle}{\partial T},$$

satisfies

$$C_V = \frac{\beta}{T} [\langle H^2 \rangle - \langle H \rangle^2],$$

i.e. indicates the fluctuations around the average potential value.

3. Compute the thermodynamic energy (average potential), specific heat, and entropy of a decoupled ($J = 0$) Ising system with N spins at temperature T and external field h.

4. Same as problems 2 and 3, but with respect to *magnetic susceptibility*

$$\chi = \frac{\partial \langle M \rangle}{\partial h} = \beta [\langle M^2 \rangle - \langle M \rangle^2]$$

in place of specific heat.