T-79.5201 Discrete Structures, Autumn 2006

Tutorial 8, 29 November

1. As is well known, the ordinary generating function of the Fibonacci numbers is $f(z) = z/(1 - z - z^2)$. Derive from this fact an estimate for the size of the Fibonacci numbers f_n, $n \geq 0$, based on information about the poles of the function $f(z)$.

2. The exponential generating function of the Bernoulli numbers is $\hat{b}(z) = z/(e^z - 1)$. Derive from this fact an estimate for the size of the numbers b_n. How precise can you make your estimate?

3. Theorem 7.1 of the lecture notes, concerned with estimating the coefficients of meromorphic generating functions, claims that if function $f(z) = \sum_{n \geq 0} f_n \, z^n$ has a pole of order m at $z_0 \neq 0$, then its contribution to the coefficient f_n is

$$-\operatorname*{Res}_{z=z_0} \frac{f(z)}{z^{n+1}} = (\frac{1}{z_0})^n \cdot P(n),$$

   where $P(n)$ is a polynomial of degree $m - 1$. Prove this claim (i.e. the fact that the residue is of the required form) when (a) $m = 1$, (b) $m \geq 1$. In the case $m = 1$ verify also the explicit formula given for the polynomial (which in this case is just a constant), $P = -\mathrm{Res}(f; z_0)/z_0$. (*Hint:* If you wish, you can follow the derivation given in H. Wilf's book *generatingfunctionology*, page 174.)