

T-79.5201 Discrete Structures, Autumn 2006

Home assignment 3 (due 20 Dec at 4 p.m.)

1. Estimate, up to an order you consider appropriate, the asymptotic growth rates of the coefficient sequences of the following generating functions. Determine your estimates directly from the analytical properties of the functions, without solving the coefficient sequences explicitly.

(a) ogf $a(z) = \frac{1}{(1+2z)(1-2z)^2}$.

(b) egf $\hat{f}(z) = ze^{z^2}$,

2. In problem 3 of tutorial 5 exponential generating functions were derived for families of permutations where (a) all the cycles are of length three ($\hat{a}(z) = e^{z^3/3}$) and (b) all the cycles are of even length ($\hat{b}(z) = (1 - z^2)^{-1/2}$). Estimate the number of n -element permutations of types (a) and (b), directly from the respective egf's without solving their coefficients explicitly.
3. Estimate, up to a precision you consider appropriate, the number of n -bit strings consistent with the regular expression

$$(a \cup b)^* d (a \cup b \cup c)^* d (a \cup b)^*$$

Derive your estimate directly from the analytical properties of the respective generating function, without solving its coefficients explicitly.

4. Estimate, using Darboux's theorem, the number of n -bit strings generated by the following context free grammars:
 - (a) $S \rightarrow aSS \mid b \mid c$
 - (b) $S \rightarrow aSS \mid bS \mid c \mid d$