Instructions

- Remember to justify your answers, and be precise. A good way of replying is to explain your solution as you would to a B.Sc. student studying CS at TKK.
- The problems are to be solved *individually* – *no plagiarism* is tolerated. If you discuss the exercises with other students, please mention their names.

You can hand in your answers by

- handing them to the lecturer at lectures, or
- slipping them into the post box in-between rooms B336 and B337, or
- sending them by e-mail to the assistant (`mjj[AT]tcs.tkk.fi`). In this case the only acceptable file formats are postscript, pdf, and ASCII text. Also, please start the Subject-header with "T-79.5103".

Write all of the following information on the first sheet:

- Course code, name, and year
  "T-79.5103 Computational Complexity Theory 2007"
- Your name and student ID
- Home assignment round (e.g., HA 1)

If you return answers on paper:

- Use paper size A4 only
- If you return multiple sheets, bind them together with e.g. staples
- Please take copies of your sheets in case you want to save them.

If you return answers by email:

- Use the filename lastname-homeassignmentround.fileformat.
  For example: `jarvisalo-1.ps`
- Only file formats ps, pdf, and txt are allowed

Reviewing and Revising Your Answers

You can get feedback on your solution with the assistant on 5.11. right after the tutorial session. If you get less than 1.5 points for some exercise, you then have the possibility to revise your answer. The **deadline for submitting revised answers is 12.11.2007**. Revised answer will be graded using the scale $0 - 1.5$.

Deadline for these exercises is 24.10.2007

Exercises

1.1 Decidability (0-2 points)

Show that the following problems are undecidable.

(i) Given a Turing machine M and a nonnegative number n in binary, does M halt on n strings? (i.e., are there n strings for which M halts?)
(ii) Given two Turing machines M,M' and a string s, is $L(M) = L(M') \cup \{s\}$?

1.2 Properties of P and NP (0-2 points)

Show that (i) **P** and (ii) **NP** are closed under union and intersection.

1.3 Reductions and Completeness (0-2 points)

A quadratic-time reduction R is computed by a deterministic Turing machine which for all inputs x halts after $O(|x|^2)$ steps with output $R(x)$. Prove that there are no **P**-complete problems under quadratic-time reductions. (Hint: Such a problem would be in TIME(n^k) for some k.)

1.4 Basic Complexity Classes and Reductions (0-2 points)

Prove that the classes **P**, **NP**, **L**, **NL**, **PSPACE**, and **EXP** are closed under (log-space) reductions. Is TIME(n^3) closed under reductions?

1.5 Generic Complete Problems (0-2 points)

Show that all languages in TIME($f(n)$) reduce to

$$\{M; x \mid M \text{ accepts } x \text{ in } f(|x|) \text{ steps}\},$$

where $f(n) > n$ is a proper complexity function.