

1. Determine the least model for the following positive program P :

$$\begin{array}{ll} p_1 \leftarrow q_1. & r. \\ q_1 \leftarrow p_1. & s_1 \leftarrow p_1, q_1. \\ p_2 \leftarrow q_2. & s_2 \leftarrow p_2, q_2. \\ q_2 \leftarrow p_2. & q_1 \leftarrow r. \end{array}$$

Which atoms of $\text{Hb}(P)$ are logical consequences of P ? Provide a counter model for one of the atoms which is not a logical consequence of P .

2. Consider the positive program P consisting of three rules

$$Q(f(x)) \leftarrow Q(x), R(g(x)). \quad R(g(x)) \leftarrow R(x). \quad R(a).$$

involving variables. What is the least Herbrand model of P ?

3. Consider the following positive program P :

$$\begin{array}{l} D(x) \leftarrow A(x), B(x). \quad E(x, y) \leftarrow D(x), D(y). \\ A(1). \quad A(2). \quad B(2). \quad B(3). \quad C(4). \end{array}$$

- (a) Form the ground program $\text{Gnd}(P)$.
- (b) Calculate the unique answer set $\text{LM}(\text{Gnd}(P))$ of P .
- (c) Use it to determine answer substitutions for the query $\exists x E(2, x)$.
- (d) Find the least subset $Q \subseteq \text{Gnd}(P)$ for which

$$\text{LM}(Q) = \text{LM}(\text{Gnd}(P))$$

holds, i.e., the rules in $\text{Gnd}(P) \setminus Q$ can be deemed redundant.

- (e) Use the `smodels` system to compute $\text{LM}(\text{Gnd}(P))$. Does the output of `lparse` differ much from Q ?