**T-79.5102** **Autumn 2006**
**Special Course in Computational Logic**
**Tutorial 4**

1. Investigate the complexity of exact inference in general Bayesian networks.

   Prove that any instance of the 3-SAT problem

   *Is the given set of 3-literal clauses $C$ satisfiable?*

   can be reduced to exact inference in a Bayesian network $N(C)$ constructed to represent the particular problem instance $C$.

   This indicates that exact inference in Bayesian networks in NP-hard.

   (R&N, Exercise 14.8.a)

2. Consider the problem of generating a random sample from a specified distribution on a single variable. You can assume that a random number generator available that returns a random number uniformly distributed between 0 and 1.

   (a) Let $X$ be a discrete variable with $P(X = x_i) = p_i$ for $i \in \{1, ...., k\}$. The **cumulative distribution** of $X$ gives the probability that $X \in \{x_1, \ldots, x_j\}$ for each possible $j$. Explain how to calculate the cumulative distribution in $O(k)$ time and how to generate a single sample of $X$ from it.

   Can the latter be done in less than $O(k)$ time?

   (b) Now suppose that we want to generate $N$ samples of $X$ where $N \gg k$. Explain how to do this with an expected runtime per sample that is *constant* (i.e., independent of $k$).

   (R&N, Exercise 14.9.ab)