

1. Consider following probability distribution for Boolean variables *Toothache*, *Cavity* and *Catch*.

	<i>toothache</i>		$\neg$ <i>toothache</i>	
	<i>catch</i>	$\neg$ <i>catch</i>	<i>catch</i>	$\neg$ <i>catch</i>
<i>cavity</i>	0.108	0.012	0.072	0.008
$\neg$ <i>cavity</i>	0.016	0.064	0.144	0.576

Calculate following probabilities based on the distribution:

- (a)  $P(\textit{toothache})$
  - (b)  $P(\textit{toothache}|\textit{cavity})$
  - (c)  $P(\textit{cavity})$
  - (d)  $P(\textit{cavity}|\textit{toothache} \vee \textit{catch})$
2. After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease, and that the test is 99% accurate (i.e., the probability of testing positive given that you have the disease is 0.99, as is the probability of testing negative given that you don't have the disease). The good news is that this is a rare disease, striking only one in 10000 people. Why is it good news that the disease is rare? What are the chances that you actually have the disease? (R&N, Exercise 13.8)
3. Use the axioms of probability to derive an expression for  $P(\phi \leftrightarrow \neg\psi)$  which is solely based on probabilities  $P(\phi)$ ,  $P(\psi)$ , and  $P(\phi \wedge \psi)$ . Note that  $\phi$  and  $\psi$  above are arbitrary propositional sentences.
4. Three prisoners, A, B, and C, are locked in their cells. It is common knowledge that one of them will be executed the next day and the others pardoned. Only the governor knows which one will be executed. Prisoner A asks the guard a favor: "Please ask the governor who will be executed, and then take the a message to one of my friends B and C to let him know that he will be pardoned in the morning." The guard agrees, and comes back later and tells A that he gave the pardon message to B. (R&N, Exercise 13.16)  
 What are A's chances of being executed, given this information?