T-79.4501
Cryptography and Data Security

Lecture 5:
5.1 MAC-functions
5.2 Hash-functions
Stallings: Ch 11, Ch 12

5.1.Message authentication codes (MAC)

(Secret key , Message) ⟶ MAC

- A MAC of a message P of arbitrary length is computed as a function $H_K(P)$ of P under the control of a secret key K. The MAC is appended to the message by the sender.
- Given a message P and its MAC value M, the MAC can be verified by anybody in possession of the secret key K and the MAC computation algorithm.
- The MAC length m is fixed.
- Security requirement: it must be infeasible, without the knowledge of the secret key, to determine the correct value of $H_K(P)$ with a success probability larger than $1/2^m$. This is the probability of simply guessing the MAC value correctly at random. It should not be possible to increase this probability even if a large number of correct pairs P and $H_K(P)$ is available to the attacker.

Example: A Weak MAC

E_K is an encryption function of a block cipher

Given a message $P = P_1, P_2, \ldots, P_n$

a MAC is computed as

$$H_K(P) = E_K(P_1 \oplus P_2 \oplus \ldots \oplus P_n)$$

Then it is easy to produce a different message P' with an equal MAC without knowledge of the secret key:

$$P' = P'_1, P'_2, \ldots, P'_{n-1}, \left(\bigoplus_{i=1}^{n-1} P'_i \right) \oplus \left(\bigoplus_{i=1}^{n} P_i \right)$$

Derived security requirements

The requirement: It must be infeasible, without the knowledge of the secret key, to determine the correct value of $H_K(P)$ for any plaintext P with a success probability larger than $1/2^m$.

This means, in particular, that the following are satisfied

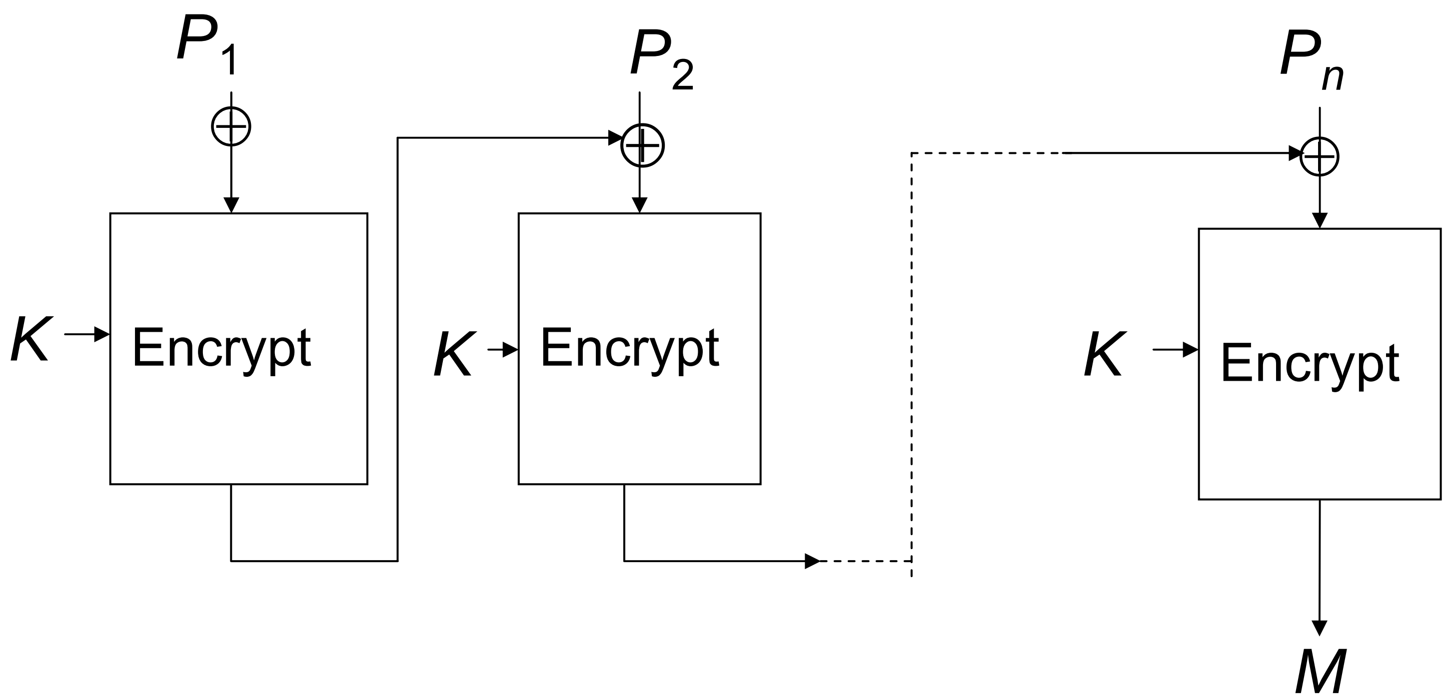
- Given a message P and $M = H_K(P)$ it should be infeasible to produce a modified message P' such that $H_K(P') = M$ without the knowledge of the key
- The function $P, K \rightarrow H_K(P)$ is one-way, that is, given a MAC value M, it should be infeasible to find a message P and a key K such that $H_K(P) = M$.
- Given known MACs for a number of known (or chosen or adaptively chosen) messages, it should be infeasible to derive the key.

MAC Designs

- Similarly as block ciphers, MAC algorithms operate on relatively large blocks of data.
- Most MAC functions are iterated constructions.
  - The core function of the MAC algorithm is a so called compression function.
  - At each round the compression function takes a new data block and compresses it together with the compression result from the previous rounds.
  - The length of the message to be authenticated determines how many iteration rounds are required to compute the MAC value.

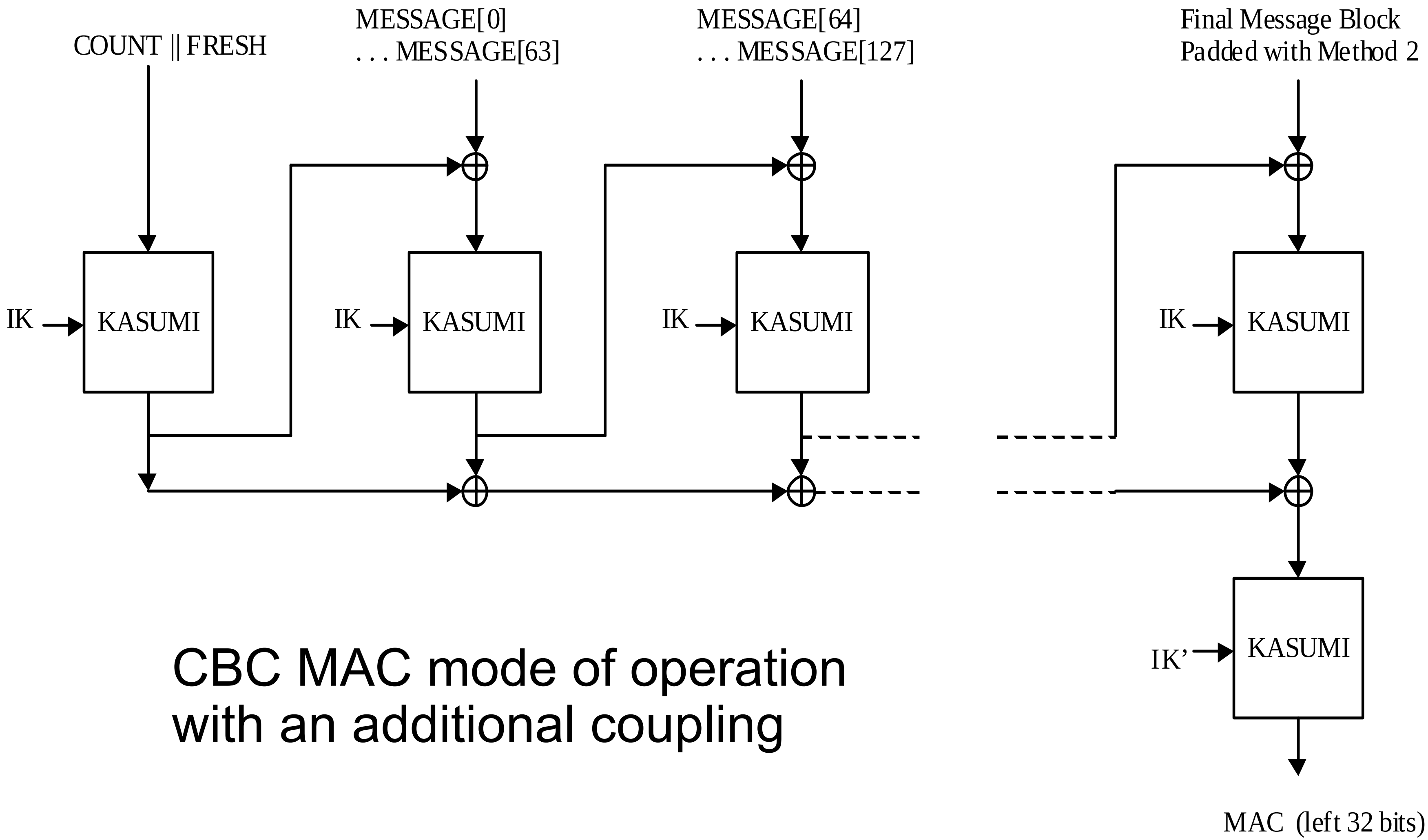
CBC MAC

A MAC mode of operation of any block cipher



- CBC encryption with fixed IV = `00…0`. The last ciphertext block (possibly truncated) is taken as the MAC.

Integrity function f9



CBC MAC mode of operation
with an additional coupling

CRC MAC

- A MAC for stream ciphers (see HAC 9.5.4.)
- Idea: A simple (cryptographically insecure) error detecting check sum is encrypted using non-repeating keystream (ideally, a one-time pad)

An n-bit message $P = p_0, p_1, \ldots, p_{n-1}$ is associated with the polynomial

$$P(x) = p_0 + p_1 x + p_2 x^2 + \ldots + p_{n-1} x^{n-1}$$

The secret key K consists of a polynomial $q(x)$ of degree m, and an m-bit one-time key stream string $(k_0, k_1, k_2, \ldots, k_{m-1})$.

First the remainder $c_0 + c_1 x + c_2 x^2 + \ldots + c_{m-1} x^{m-1}$ of the polynomial division $P(x)/q(x)$ is computed. The MAC is computed as the xor of the key stream string and the remainder string $(c_0, c_1, c_2, \ldots, c_{m-1})$ as

$$(c_0 \oplus k_0, c_1 \oplus k_1, c_2 \oplus k_2, \ldots, c_{m-1} \oplus k_{m-1})$$

Note: The polynomial $q(x)$ can be reused for different messages

Polynomial MAC

- Another MAC suitable for use with stream ciphers
- Idea: An (cryptographically insecure) error detecting code is encrypted using non-repeating keystream (ideally, a one-time pad)
- An n-block message $P = P_0, P_1, \ldots, P_{n-1}$ with block size m bits is associated with the polynomial with m-bit coefficients $P_i \in \mathrm{GF}(2^m)$:

$$P(x) = P_0 + P_1 x + P_2 x^2 + \ldots + P_{n-1} x^{n-1}$$

- The value of the polynomial taken in point $x \in \mathrm{GF}(2^m)$ is taken as an m-bit string $P(x) \in \mathrm{GF}(2^m)$.
- The secret key K consists of a point $x = X$ and an m-bit one-time key stream string $(k_0, k_1, k_2, \ldots, k_{m-1})$.
- First the message polynomial is evaluated at the point x = X. Let us denote $P(x) = (c_0, c_1, c_2, \ldots, c_{m-1})$. The MAC is computed as the xor of the key stream string and the value as

$$H_K(P) = (c_0 \oplus k_0, c_1 \oplus k_1, c_2 \oplus k_2, \ldots, c_{m-1} \oplus k_{m-1})$$

Note: The point X can be reused for different messages

An Example

Poly1305-AES MAC

- By D J Bernstein, presented at FSE2005, http://cr.yp.to/mac.html
- Over finite fields: Carter-Wegman MAC and Galois MAC (with Counter Mode key stream generator)

Combined modes of operation

Authenticated Encryption Modes

- CCM: Counter mode encryption and CBC MAC , see:

  1) IETF RFC 3610

  2) NIST Special Publication SP800-38C (with consideration to the IEEE 802.11i)
- GCM: Counter mode encryption and a Polynomial-MAC over Galois Field, see: http://csrc.nist.gov/CryptoToolkit/modes/proposedmodes/

5.2 Hash functions

Message $\longrightarrow$ Hash code

- A hash code of a message *P* of arbitrary length is computed as a function *H*(*P*) of *P* . The hash length *m* is fixed.
- Hash function is public: Given a message *P* anybody can compute the hash code of *P.*
- Security requirements:
  1. *Preimage resistance:* Given *h* it is impossible to find *P* such that *H*(*P*) = *h*
  2. *Second preimage resistance:* Given *P* it is impossible to find *P'* such that *H*(*P'*) = *H*(*P*)
  3. *Collision resistance:* It is impossible to find *P* and *P'* such that *P* ≠ *P'* and *H*(*P'*) = *H*(*P*)

Collision Attack

- An upperbound to the security of hash functions is due to the collision probability which is estimated using Birthday Paradox.
- Random oracle: Given a message an ideal hash function, with m-bit output, computes the hash value by selecting the value uniformly at random from all possible 2^m values. To find a collision with probability at least 1/2, approximately $1{,}17 \cdot 2^{m/2}$ messages need to be hashed. This gives an estimate to the workload of making the collision attack successful.

Design Principles

- Similarly as MAC algorithms, hash functions operate on relatively large blocks of data.
- Most hash functions are iterated constructions.
  - The core function in a hash function is a compression function.
  - At each round the compression function takes a new data block and compresses it together with the compression result from the previous rounds.
  - Hence the length of the message to be authenticated determines how many iteration rounds are required to compute the MAC value.

SHA-1

- Designed by NSA
- FIPS 180-1 Standardi 1995 –
  www.itl.nist.gov/**fips**pubs/**fip**180-1.htm

February 2005:

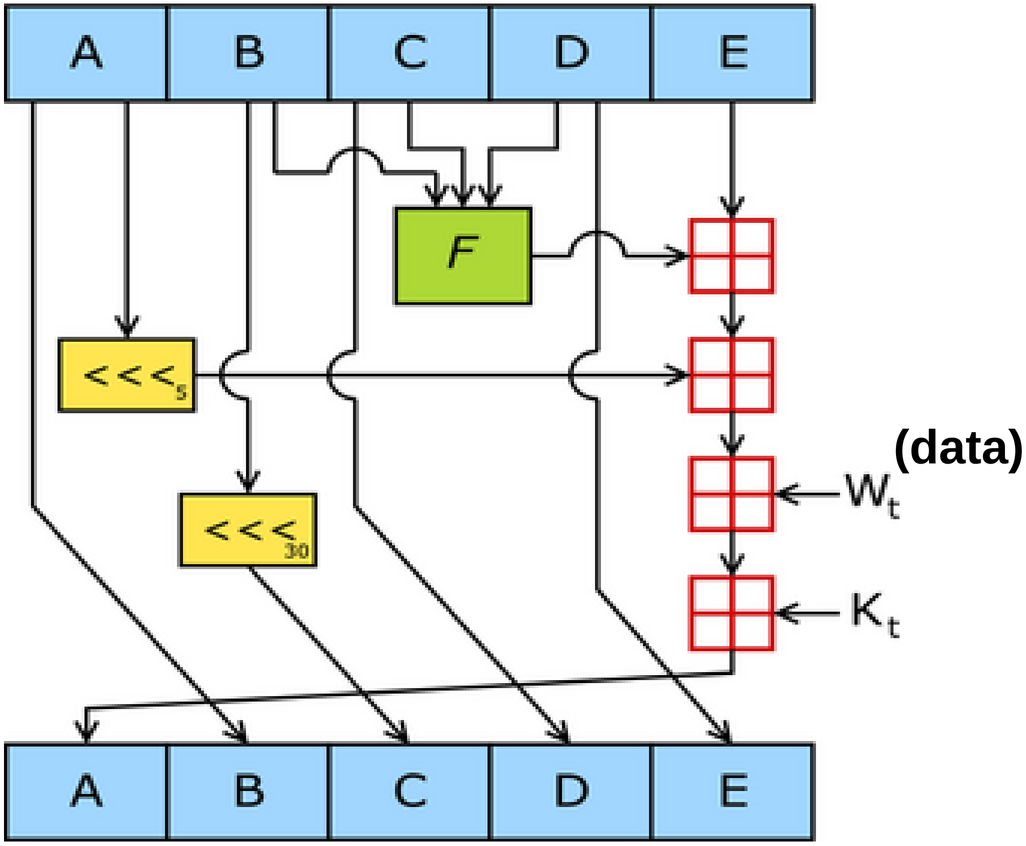
Professor Xiaoyun Wang (Shandong University) announce an algorithm which finds collisions for SHA-1 with complexity 2^{69} (today reduced to 2^{63})

Recommendation: Use 256- or 512-bit versions of SHA:
csrc.nist.gov/publications/**fips**/**fips**180-2/**fips**180-2.pdf

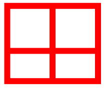
SHA-1

- Step 1: Append padding bits.
- Step 2: Append $length$ = length of the message in bits before padding (64 bits); then fin L such that $length + 64 \leq 512\ L$
- Step 3: Initialise MD buffer composed of five 32-bit registers (A,B,C,D,E) with a constant IV (fixed in the spec).This is denoted by CV_0.
- Step 4 (repeated L times): Process message in 512-bit (16-word) blocks. It takes 80 rounds. At the end, the contents of the registers ABCDE are added to the input CV_q.The addition modulo 2^{32} is done for each word separately. The result is the output CV_{q+1} (input to the next round), $q = 0,1,\ldots,L\text{-}1$. The addition modulo 2^{32} is done for each word separately.
- Step 5: Output is CV_L

SHA-1 Compression function
One round



512 bits of data – 80 rounds

 Addition modulo 2^{32}

Function F and data expansion

$$q = 0,\dots,19: \quad F_q(B,C,D) = (B \wedge C) \vee (\overline{B} \wedge D)$$

$$q = 20,\dots,39: \quad F_q(B,C,D) = B \oplus C \oplus D$$

$$q = 40,\dots 59: \quad F_q(B,C,D) = (B \wedge C) \vee (B \wedge D) \vee (C \wedge D)$$

$$q = 60,\dots 79: \quad F_q(B,C,D) = B \oplus C \oplus D$$

Data expansion:

$(W_0, W_1, W_2, \dots, W_{15}) =$ the 512-bit input data block

$$W_q = \lll_1 (W_{q-16} \oplus W_{q-14} \oplus W_{q-8} \oplus W_{q-3}), \; q = 16 \dots 79$$

Revised SHA Standard

csrc.nist.gov/publications/**fips**/**fips**180-2/**fips**180-2.pdf

	SHA-1	SHA-256	SHA-384	SHA-512
Hash size	160	256	384	512
Message size	$< 2^{64}$	$< 2^{64}$	$< 2^{128}$	$< 2^{128}$
Block size	512	512	1024	1024
Word size	32	32	64	64
Number of steps	80	80	80	80
Claimed security	2^{80}	2^{128}	2^{192}	2^{256}

HMAC- hash based MAC

- RFC 2104: the MAC for IP security
- To use available hash functions
- To allow hash function to be replaced easily
- To preserve the performance of a hash function
- Easy handling of keys
- Well understood cryptographic security

- Recent collision attacks against hash functions do not effect HMAC constructions

HMAC algorithm

H	hash function
M	message input to HMAC (after hash function specific padding added)
L	number of blocks in M
b	number of bits in a block
n	length of the hash code of H
K	secret key, recommended length ≥ n
K^+	a b-bit string formed by appending zeros to the end of K
$ipad$	= `00110110` repeated b/8 times
$opad$	= `01011100` repeated b/8 times

$$HMAC(K;M) = H[(K^+ \oplus opad) \,\|\, H[(K^+ \oplus ipad) \| M]]$$