T-79.4501 Cryptography and Data Security
2006 / Homework 2
Wed 27.9 and Thu 28.9

1. Consider the DES S-box $S_{4}$

| 7 | 13 | 14 | 3 | 0 | 6 | 9 | 10 | 1 | 2 | 8 | 5 | 11 | 12 | 4 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 13 | 8 | 11 | 5 | 6 | 15 | 0 | 3 | 4 | 7 | 2 | 12 | 1 | 10 | 14 | 9 |
| 10 | 6 | 9 | 0 | 12 | 11 | 7 | 13 | 15 | 1 | 3 | 14 | 5 | 2 | 8 | 4 |
| 3 | 15 | 0 | 6 | 10 | 1 | 13 | 8 | 9 | 4 | 5 | 11 | 12 | 7 | 2 | 14 |

(a) For the following 6 -bit inputs: $000000,010011,101100,111011$, what are the corresponding outputs?
(b) Show that the second row of $S_{4}$ can be obtained from the first row by means of the following mapping:

$$
\left(y_{1}, y_{2}, y_{3}, y_{4}\right) \mapsto\left(y_{2}, y_{1}, y_{4}, y_{3}\right) \oplus(0,1,1,0)
$$

2. Let us consider the mult operation in IDEA.
(a) Use the Extended Euclidean Algorithm to compute the inverse of 357 with respect to mult.
(b) What is the inverse of 0 with respect to mult?
3. The Mangler function of IDEA takes two 16 -bit data inputs $Y_{i n}$ and $Z_{i n}$ and it produces two 16 -bit outputs $Y_{\text {out }}$ and $Z_{\text {out }}$, and it is controlled by two 16 -bit keys $K e$ and $K e$ (see Lecture 3). Compute the outputs with the following keys and inputs:
(a) $K e=K f=1024$ and $Y_{i n}=Z_{\text {in }}=64$
(b) $K e=Z_{\text {in }}=512$ and $K f=Y_{\text {in }}=128$
4. Show that the even round of IDEA with any given round keys $K e$ and $K f$ is its own inverse.
5. In the round key expansion procedure Rijndael makes use of eight-bit constants $C_{i}$, $i=1,2,3, \ldots, 30$ that can be computed as

$$
C_{i}=2^{i-1}
$$

in polynomial arithmetic modulo $m(x)=x^{8}+x^{4}+x^{3}+x+1$. For example, $C_{1}=00000001, C_{2}=2=00000010$, etc. Compute $C_{11}, C_{12}$ and $C_{13}$.

