T–79.4301 Parallel and Distributed Systems (4 ECTS)

T–79.4301 Rinnakkaiset ja hajautetut järjestelmät (4 op)

Lecture 6

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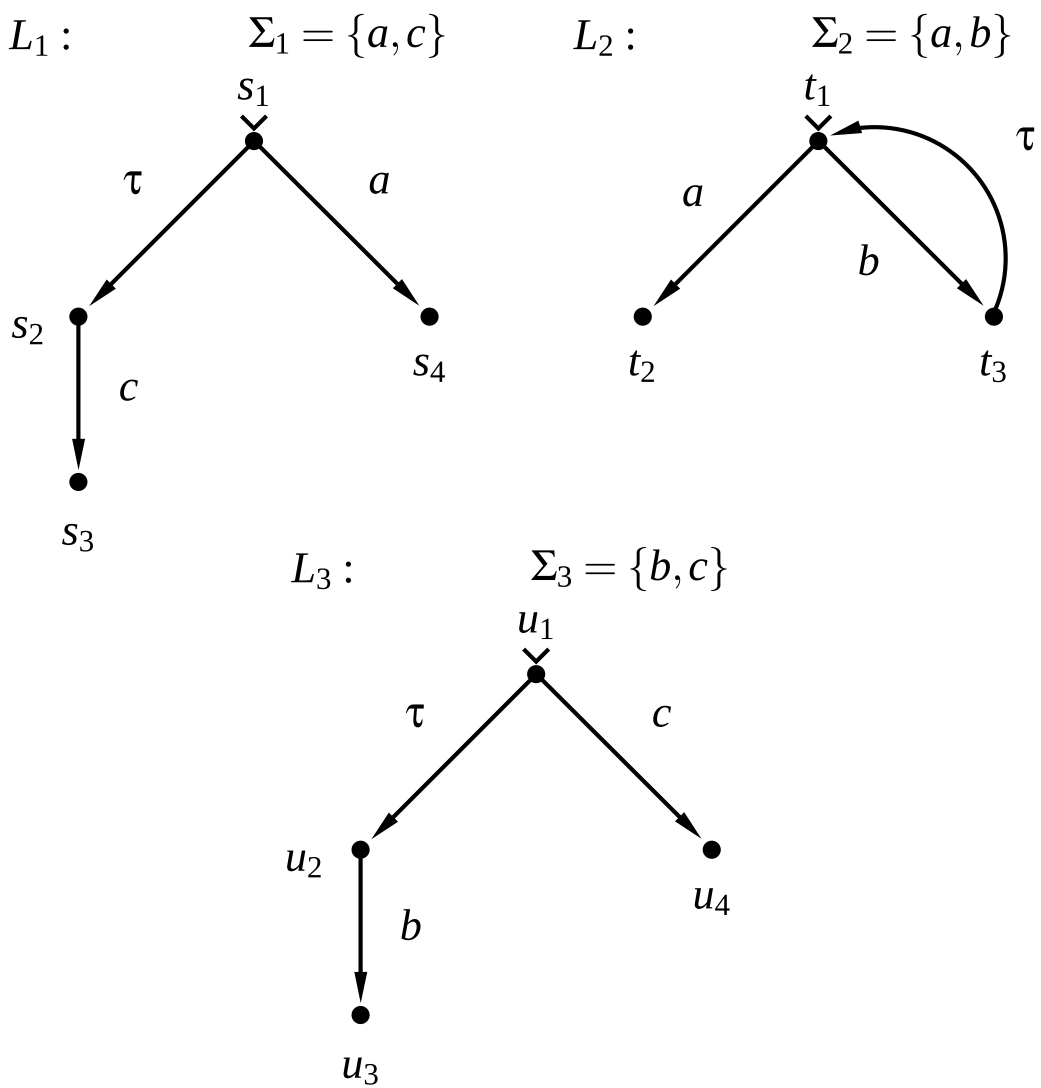
Running Example - LTSs

Consider now our running LTS example from Lecture 5, reproduced on the next slide for convenience:

- We use as global transitions tuples $t = (t_1, t_2, t_3)$, where $t_i \in \Delta_i$ or $t_i =$ "$-$" in the case the LTS i does not take part in t
- In our running example $v^0 = (s_1, t_1, u_1)$
- $\mathit{enabled}(v^0) = \{\ ((s_1, \tau, s_2), -, -),$ $((s_1, a, s_4), (t_1, a, t_2), -),\ (-, -, (u_1, \tau, u_2))\ \}$
- $\mathit{fire}((s_1, t_1, u_1), ((s_1, a, s_4), (t_1, a, t_2), -)) = (s_4, t_2, u_1)$

Running Example - LTSs (recap)



Deadlocks

Let's now formally define some new concepts for LTSs.

- A *deadlock* is a global state v in the reachability graph such that $enabled(v) = \emptyset$
- Quite often (but not always) deadlocks are signs of "bad behavior" in the analyzed system. (Some behaviors of the system might be modelling "successful" termination of the system.)

Deadlocks (cnt.)

- The process of checking whether the reachability graph contains any deadlock states is called deadlock checking, and it can be easily added to the basic reachability analysis algorithm:
  - Report a deadlock if `enabled(v)` returns the empty list of enabled transitions for some reachable state `v`.

Livelock

- A *livelock* (also called divergence) exists in a state s in an LTS L, if $s \xrightarrow{\tau} s'$ and $s' \xrightarrow{\tau^*} s$ for some state s'.
- Intuitively a livelock (divergence) corresponds to a cycle in the LTS where the LTS performs only internal τ-transitions.
- As is the case with deadlocks, quite often (but not always) livelocks are signs of bad behavior in the analyzed system.
- Livelocks need another (simple) search algorithm to be detected, they cannot be detected with a single DFS or BFS.

Conflict

- A *conflict* occurs in a reachable global state v of $L = L_1 || L_2 || \cdots || L_n$ if there are (at least) two *conflicting transitions* t and t' in $enabled(v)$ such that
  - there is an LTS L_i with $1 \leq i \leq n$, such that $t = (\ldots, t_i, \ldots)$, $t' = (\ldots, t'_i, \ldots)$, and $t_i \neq t'_i$.

In other words, in the case of a conflict there is some component i which has at least two local transitions t_i and t'_i enabled, and it can fire either, having to choose between the two possibilities.

Conflict - Intuition

- If a reachability graph of a system has no conflicts in it, all non-determinism in it is caused by scheduling speeds of components
- Intuitively, conflict free systems are “concurrent, yet fully deterministic”, i.e., their behavior contains no “true non-deterministic choices”. This simplifies their analysis greatly.
- Unfortunately all real systems have conflicts: Whenever there is a resource shared between two components in a mutex manner, a conflict is going to happen when it is allocated to either one of the two components requesting access to it

Independence

- Two global transitions $t = (t_1, t_2, \ldots, t_n)$ and $t' = (t'_1, t'_2, \ldots, t'_n)$ are *independent* iff
  - $t \neq t'$, and
  - for all $1 \leq i \leq n$: if $t_i \neq$ "$-$" then $t'_i =$ "$-$" and if $t'_i \neq$ "$-$" then $t_i =$ "$-$".

Intuitively the set of LTSs which participate in t and t' need to be disjoint for the two transitions to be independent.

Independence (cnt.)

Note that independence (as defined in this course) is a static property of global transitions (independent of the current state v), while conflicts are a dynamic property (both conflicting transitions need to be enabled in v).

Independence (cnt.)

- Two independent transitions t and t' can never be in conflict, and two conflicting transitions t and t' can never be independent.
- If two transitions t and t' are independent and they are both in $enabled(v)$ then they are said to be concurrent. In this case both of the sequences of transition t,t' and t',t can be fired from v, and the two states reached by doing so will be the same.

Independence (cnt.)

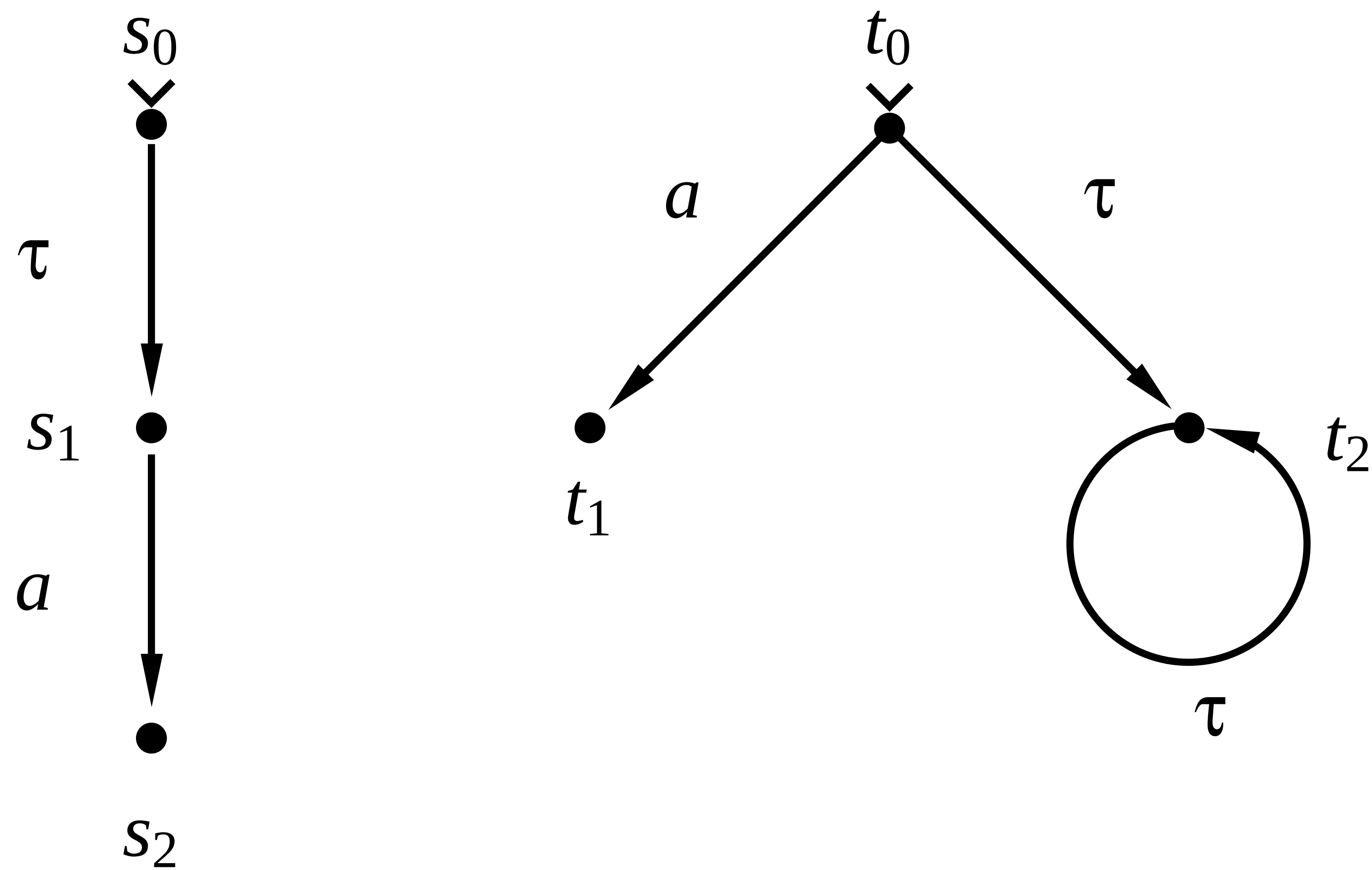
- If a state v has n pairwise independent transitions in $enabled(v)$, then any reachability graph containing v has at least 2^n nodes and $n \cdot 2^{(n-1)}$ edges. There are $n! = 2^{O(n \log_2 n)}$ possible orders of firing the independent transitions.
- Such a structure is called a “diamond”, and it can be seen as an n-dimensional hypercube (hint: 1-dimensional hypercube is the line, 2-dimensional hypercube is the square, and 3-dimensional hypercube is the cube) with a single entry vertex, where all edges are directed arcs which are directed towards a single exit vertex

Independence (cnt.)

- So called *partial order reductions* use independence between transitions to remove reachable states from the reachability graph while still preserving, e.g., the existence of deadlocks
- It is a *common beginners mistake* to assume that going through each "diamond" induced by n independent transitions at v by taking the independent transitions of $enabled(v)$ in exactly one order will preserve the existence of, e.g., deadlocks in the reachability graph. This is NOT the case!

Counterexample: Independence

$L_1:$ $\Sigma_1 = \{a\}$ $L_2:$ $\Sigma_2 = \{a\}$



Example: Independence (cnt.)

- In the initial state $v^0 = (s_0, t_0)$, $enabled(v^0) = \{((s_0, \tau, s_1), -), (-, (t_0, \tau, t_2))\}$
- Now the two enabled transitions are independent
- If we only fire $t' = (-, (t_0, \tau, t_2))$ in v^0, the deadlock state (s_2, t_1) reachable by first firing $t = ((s_0, \tau, s_1), -)$ at v^0 and then firing $t'' = ((s_1, a, s_2), (t_0, a, t_1))$ is no longer reachable

Example: Independence (cnt.)

- Thus by removing some “internal nodes of the diamond” deadlocks of the system can be missed. (Note: This phenomenon is sometimes called “confusion” in the concurrency literature.)
- The partial order reduction methods know how to deal with this problem while still being able to remove some unnecessary interleavings of independent transitions of the system

Partial Order Reductions Disabled

```
$ spin -a peterson3
$ gcc -o pan -DNOREDUCE pan.c
$ ./pan
hint: this search is more efficient if pan.c is compiled -DSAFET
(Spin Version 4.2.6 -- 27 October 2005)

Full statespace search for:
        never claim             - (none specified)
        assertion violations    +
        acceptance   cycles     - (not selected)
        invalid end states      +
```

Partial Order Reductions Disabled

```
State-vector 28 byte, depth reached 5837, errors: 0
   25362 states, stored
   44425 states, matched
   69787 transitions (= stored+matched)
       0 atomic steps
hash conflicts: 791 (resolved)

Stats on memory usage (in Megabytes):
... stuff removed ...
3.236   total actual memory usage
```

Comparison

- The partial order reductions in Spin are on by default but can be disabled by the "`-DNOREDUCE`" compile time option
- Compared to the results in Lecture 5, disabling the partial order reductions results in:
  - Number of stored states rose from 2999 to 25362
  - Number of transitions rose from 3805 to 69787
  - The effect was modest (just one order of magnitude) because the example only has three parallel processes. Usually the differences are even larger.

Ample Sets

- The partial order reduction algorithm implemented in Spin is based on a method called ample sets. (Similar methods: persistent and stubborn sets.)
- The most upto-date description of the Spin algorithm can be found from Chapter 10 of the book:
  - Edmund M. Clarke, Jr., Orna Grumberg, and Doron Peled: Model Checking, MIT Press, 1999.
  - `http://mitpress.mit.edu/book-home.tcl?isbn=0262032708`
- The algorithms inside Spin and other explicit state model checkers are a main topic of the course: T–79.5301 Reactive Systems

  `http://www.tcs.hut.fi/Studies/T-79.5301/`

Traces

- The set of traces of an LTS L is defined to be set of sequences of visible actions of L:
  $traces(L) = \{\sigma \in \Sigma^* \mid L \overset{\sigma}{\Rightarrow}\}$.
  (Recall: $\tau \notin \Sigma$.)
- Intuitively: $traces(L)$ is the language of all executions of L projected on Σ, thus removing all τ-transitions.
- Another intuition: See L as a non-deterministic FSA $A = lts2fsa(L)$ where all τ-transitions have been replaced with ε-moves, and all states are accepting. Now $traces(L)$ is the language accepted by A.

Traces (cnt.)

- Two LTSs L_1 and L_2 are called trace equivalent iff $traces(L_1) = traces(L_2)$.
- The trace preorder $\leq_{tr}$ is defined as follows: $L_1 \leq_{tr} L_2$ iff $traces(L_1) \subseteq traces(L_2)$.
- Hint: A preorder is a just a relation which is reflexive and transitive.

Traces (cnt.)

- An LTS L deadlocking in the initial state has $traces(L) = \{\varepsilon\}$ (where ε denotes the empty word), and therefore $L \leq_{tr} L'$ for any L'.
- An LTS L with $traces(L'') = \Sigma^*$ is the maximal element of the trace preorder, i.e., $L' \leq_{tr} L''$ for any LTS L'.
  - It is easy to construct L'' such that $traces(L'') = \Sigma^*$: the LTS has one state s^0, and a transition $s^0 \xrightarrow{a} s^0$ for all $a \in \Sigma$.

Checking Trace Containment

- To check whether $L \leq_{tr} L'$ we proceed as follows:
  - Create a FSA $A' = lts2fsa(L')$.
  - Create a FSA $A'' = det(A')$, the determinized version of A' with a total transition relation. (Requires changing the definition of $det(\cdot)$ slightly to also handle ε-moves.)
  - Create a FSA $A''' = \overline{A''}$ by swapping the final and non-final states of A''.

Checking Trace Containment (cnt.)

- To check whether $L \leq_{tr} L'$ (cnt.):
  - See A''' as an LTS L'''.
  - Compute the product $P = L || L'''$.
  - Check if any state (s,t) is reachable in P, where t is a final state of A'''.
    - No: $L \leq_{tr} L'$ holds.
    - Yes: $L \leq_{tr} L'$ does not hold, and there is a sequence $\sigma \in \Sigma^*$ such that $(s^0, t^0) \overset{\sigma}{\Rightarrow} (s,t)$, and thus σ is a sequence in $traces(L)$ which does not exist in $traces(L')$.

FSA Determinization with ε-moves

Definition 1 Let $A_1 = (\Sigma, S_1, S_1^0, \Delta_1, F_1)$ be a non-deterministic automaton with ε-moves. We define a deterministic automaton $A = (\Sigma, S, S^0, \Delta, F)$, where

- $S = 2^{S_1}$, the set of all sets of states in S_1,
- $S^0 = \{s' \mid s \xrightarrow{\varepsilon^*} s', \text{ for some } s \in S_1^0\}$,
- $(Q, a, Q') \in \Delta$ iff $Q \in S, a \in \Sigma$, and $Q' = \{s'' \in S_1 \mid \text{ there is } (s, a, s') \in \Delta_1 \text{ such that } s \in Q \text{ and } s' \xrightarrow{\varepsilon^*} s''\}$; and
- $F = \{s \in S \mid s \cap F_1 \neq \emptyset\}$, those states in S which contain at least one accepting state of A_1.

Checking Trace Containment (cnt.)

- Typical application: $I \leq_{tr} S$, where I is an implementation and S is a specification.
- To check the trace containment $I \leq_{tr} S$ one has to determinize S.
- As usual, determinization requires worst-case exponential space in S (hopefully the specification S is relatively small).
- Trace containment is one of the most often used ways of checking properties of systems modelled with LTSs.

Abstraction with Traces

- Quite often we do not have resources to directly check that $I \leq_{tr} S$, because the parallel composition $I = L_1 || L_2 || \cdots || L_n$ is just too big to handle.
- We can often discard unnecessary detail from the implementation by creating some component L'_i such that $L_i \leq_{tr} L'_i$.
- Now if $L_i \leq_{tr} L'_i$ then it can be proved that also $I \leq_{tr} I'$, where I' is I with the component L_i replaced with L'_i.

Abstraction (cnt.)

- Now clearly, if $I' \leq_{tr} S$ then also $I \leq_{tr} S$.
- Thus when using trace containment as the way of checking properties, any component of the implementation can be replaced with another one provided that the new component “has more behavior” than the original.
- Hopefully the new component is smaller than the original one, leading to hopefully small I'.
- This is called abstraction: leaving out unnecessary detail by, e.g., replacing data dependent if-then-else constructs of the modelling language with purely non-deterministic choice.

Abstraction (cnt.)

Examples of abstraction in LTSs preserving traces:

- Some component L_i might be removed altogether by replacing it with the one-state component L'_i, such that $traces(L'_i) = \Sigma^*$.
- Sequences of τ-transitions can be compressed away in many cases, as long as their firing cannot be indirectly observed in the traces of the component.
- In the LTS domain in particular, it is always safe to add arcs to LTSs as doing so can only increase the set of traces of the component.

Abstraction (warning)

- Note: The set of allowed abstractions depends on the fact that we are using trace containment to check properties!
- A completely different set of allowed abstractions applies if we were, e.g., checking the implementation for deadlock freedom.
- Thus the set of modelling abstractions that are sound depends very closely on the properties that need to be verified from the model!
- Trace containment allows for more freedom in choosing the right abstraction than most other preorders.