T-79.4301 Spring 2006

Parallel and Distributed Systems Tutorial 1 – Solutions

1. a) Dekker's algorithm can be modelled in Promela as follows (the line numbers are not strictly part of the code)

```
bool flag[2] = false;
 2
    bool turn = 0;
 3
 4 active [2] proctype mutex()
 5
    {
 6
      /* i is my index, j is the other process */
 7
      pid i = pid; pid j = 1 - pid;
      /* Infinite loop */
 8
 9
    again:
10
      /* [noncritical section] */
11
      flag[i] = true;
      /* [trying section] */
12
13
      do
14
      :: flag[j] ->
15
           if
           :: turn == j ->
16
17
                           flag[i] = false;
18
                           (turn != j);
19
                           flag[i] = true;
20
           :: else -> skip
21
           fi;
22
      :: else -> break
23
      od;
24
25
      /* [critical section] */
      turn = j;
26
27
      flag[i] = false;
28
      goto again;
29
   }
```

The simple pseudo-code translates into Promela in a very straightforward way. Line 1 initialises all elements of the flag array (shared between all processes) to false. Basically, line 4 instructs the model checker Spin to analyse two concurrent processes executing the code

enclosed in the curly braces. The model checker Spin numbers these processes starting from zero; an individual process can use the special variable _pid to find its numeric identifier (line 7). The variables i and j defined at line 7 are local to each running process.

In the rest of the code, the comments, variable assignments, and tests on variables are replaced with their Promela equivalents (Promela uses C-style comments, assignments, and operators). Unlike C programs, however, where every statement reached by the program control can be "executed immediately", Promela processes may block on certain statements of the code. This feature is used at line 18 of the Promela code to implement the idle loop in the pseudo-code: a process that reaches this line of the Promela code will not proceed until the condition on the variables becomes true. To avoid similar blocking in the do-loop and the if-selection, we have to explicitly specify what the process should do when the condition at line 14 or 16 is false. The skip statement at line 20 is a no-op; a break statement can be used to break out of the innermost do-loop (line 22).

1. b) To check whether Dekker's algorithm solves the mutual exclusion problem correctly, we add to the program a new variable count (shared between the processes) that gives the number of processes that are currently in their critical section. A process increments and decrements the value of this variable whenever entering and leaving the critical section, respectively. The requirement that only one process should be in the critical section at any one time is represented by the assertion at line 27.

```
1
   bool flag[2] = false;
   bool turn = 0;
3
   byte count = 0;
4
5
   active [2] proctype mutex()
6
7
        /* i is my index, j is the other process */
8
        pid i = pid; pid j = 1 - pid;
9
        /* Infinite loop */
    again:
10
11
        /* [noncritical section] */
12
        flag[i] = true;
        /* [trying section] */
13
14
        do
```

```
15
        :: flag[j] ->
16
              if
17
              :: turn == j ->
18
                              flag[i] = false;
19
                              (turn != j);
                              flag[i] = true;
20
21
              :: else -> skip
22
              fi:
23
        :: else -> break
24
        od;
25
26
        count++;
27
        assert(count == 1);
        /* [critical section] */
28
29
        count--;
30
31
        turn = j;
32
        flag[i] = false;
33
        goto again;
34 }
```

1. c) Suppose that the Promela code from b) is in the file mutex.pml in the current directory. We can now use the model checker Spin to generate an analyser for this model by giving the command

```
$ spin -a mutex.pml
```

This command generates a C source code file (pan.c, short for "protocol analyser") along with some auxiliary files in the current directory. We then compile the file and run the analyser:

```
State-vector 20 byte, depth reached 49, errors: 0
166 states, stored
156 states, matched
322 transitions (= stored+matched)
0 atomic steps
hash conflicts: 0 (resolved)

2.622 memory usage (Mbyte)

unreached in proctype mutex
line 30, state 22, "-end-"
(1 of 22 states)
```

Intuitively, the analyser generated by Spin checks whether there exists a way to interleave the invidivual statements of the code between the two processes competing for the permission to enter the critical section such that the assertion added to the code is violated. Such an interleaving of the statements exists if and only if it is possible for the two processes to simultaneously enter their critical section. The report given by the analyser shows that no such interleaving exists. Therefore, we can conclude that the Promela model for Dekker's algorithm correctly solves the mutual exclusion problem for two processes.

1. d) To check whether the algorithm remains correct when the do-loop in the model is changed to an if-selection, we model the modified algorithm in Promela:

```
bool flag[2] = false;
2 bool turn = 0;
3 byte count = 0;
4
   active [2] proctype mutex()
5
6
7
        /* i is my index, j is the other process */
        pid i = pid; pid j = 1 - pid;
8
9
        /* Infinite loop */
10
   again:
11
        /* [noncritical section] */
12
       flag[i] = true;
13
        /* [trying section] */
```

```
14
        if
15
        :: flag[j] ->
16
             if
17
             :: turn == j ->
18
                             flag[i] = false;
                             (turn != j);
19
                             flag[i] = true;
20
21
             :: else -> skip
22
             fi;
23
        :: else -> skip
24
        fi;
25
26
        count++;
27
        assert(count == 1);
28
        /* [critical section] */
29
        count--;
30
31
        turn = j;
32
        flag[i] = false;
33
        goto again;
34 }
Repeating the analysis for this model (mutex_2.pml) gives the following
result:
$ spin -a mutex_2.pml
$ cc -o pan pan.c
$ ./pan
hint: this search is more efficient if pan.c is compiled -DSAFETY
pan: assertion violated (count==1) (at depth 104)
pan: wrote mutex_2.pml.trail
(Spin Version 4.2.6 -- 27 October 2005)
Warning: Search not completed
        + Partial Order Reduction
Full statespace search for:
        never claim
                                 - (none specified)
        assertion violations
        acceptance cycles
                                 - (not selected)
        invalid end states
```

```
State-vector 20 byte, depth reached 115, errors: 1
162 states, stored
81 states, matched
243 transitions (= stored+matched)
0 atomic steps
hash conflicts: 0 (resolved)
```

2.622 memory usage (Mbyte)

In this case the assertion is violated. Indeed, both processes can enter the critical section at the same time in this case. A simple scenario in which this occurs proceeds as follows:

- Process 1 begins its execution and sets flag[1] to true (line 12). Because flag[0] was initialised to false, only the else branch of the outer if-selection is executable. Therefore, process 1 increments count to 1 (line 26) and proceeds to the critical section.
- While process 1 is still in its critical section, process 0 begins its execution. After setting flag[0] to true (line 12), it finds flag[1] true at line 15 and proceeds to the inner if-selection. Because process 1 has not left its critical section, however, the variable turn still has its initial value 0 at this point. Therefore also process 0 is allowed to enter its critical section, and the assertion at line 27 is violated.

(In practice, counterexamples such as these can be generated automatically by the analyser. These features of the tool will be demonstrated later during the course.)

1. e) The Promela model of Peterson's algorithm (augmented with an assertion to check its correctness) is as follows:

```
/* Peterson's mutex algorithm, two parallel processes 0 and 1 */
bool flag[2] = false;
bool turn = 0;
byte count = 0;

/* flag is initialized to all false, */
/* and turn has the initial value 0 */
```

```
active [2] proctype peterson()
    pid i = pid; pid j = 1 - pid;
    /* Infinite loop */
again:
    /* [noncritical section] */
    flag[i] = true;
    /* [trying section] */
    turn = i;
    (flag[j] == false || turn != i);
    count++;
    assert(count == 1);
    /* [critical section] */
    count--;
    flag[i] = false;
    goto again
}
We again analyse the model using the Spin model checker:
$ spin -a peterson.pml
$ cc -o pan pan.c
$ ./pan
hint: this search is more efficient if pan.c is compiled -DSAFETY
(Spin Version 4.2.6 -- 27 October 2005)
        + Partial Order Reduction
Full statespace search for:
       never claim
                                - (none specified)
        assertion violations
        acceptance cycles
                               - (not selected)
        invalid end states
State-vector 20 byte, depth reached 22, errors: 0
      38 states, stored
      45 states, matched
      83 transitions (= stored+matched)
      0 atomic steps
```

```
hash conflicts: 0 (resolved)

2.622 memory usage (Mbyte)

unreached in proctype peterson
line 27, state 9, "-end-"
(1 of 9 states)
```

No errors are reported by the analyser. Therefore, the model of Peterson's algorithm correctly solves the mutual exclusion problem for two processes.