## Tutorial 5, 18 October

Problems

1. Give all solutions to the following constraint satisfaction problem (CSP)

$$
\begin{aligned}
& \left\langle\left\{C_{1}(z, y), C_{1}(y, x), C_{1}(x, z)\right\} ;\right. \\
& \quad x \in\{1,2,3\}, y \in\{1,2,3\}, z \in\{1,2,3\}\rangle
\end{aligned}
$$

where $C_{1}=\{(1,3),(1,2),(1,1),(2,3),(2,2),(3,3)\}$
2. Encode the SET COVER problem as a constraint satisfaction problem (CSP).
3. Encode the TSP optimization problem as a constrained optimization problem.
4. a) Give a propositional formula that expresses the Boolean function that the circuit below computes.

b) Give a propositional formula in CNF that expresses the Boolean function
i) $\operatorname{odd}\left(x_{1}, x_{2}, x_{3}\right)$ which evaluates to true iff an odd number of $x_{1}, x_{2}, x_{3}$ have the value true;
ii) atleast ${ }_{2}\left(x_{1}, \ldots, x_{n}\right)$ which evaluates to true iff the number of $x_{1}, \ldots, x_{n}$ having the value true is at least 2 ;
iii) atmost $_{n-1}\left(x_{1}, \ldots, x_{n}\right)$ which evaluates to true iff the number of $x_{1}, \ldots, x_{n}$ having the value true is at most $n-1$;
5. Consider the Boolean circuit given as a system of equations:

$$
\begin{aligned}
& x_{1}=\operatorname{or}\left(y_{1}, y_{2}, y_{3}\right) \\
& y_{1}=\operatorname{and}\left(z_{11}, z_{12}\right) \\
& y_{2}=\operatorname{and}\left(z_{21}, z_{22}\right) \\
& y_{3}=\operatorname{and}\left(z_{31}, z_{32}\right)
\end{aligned}
$$

Write (i) the corresponding equivalent propositional formula, (ii) give the formula in equivalent conjunctive normal form and (iii) write the corresponding CNF of the circuit directly using the Tseitin translation.

