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T-79.4201 Search Problems and Algorithms (4 cr)
Exam Mon 15 May 2006, 9–12 a.m.

Write down on each answer sheet:
- Your name, department, and study book number
- The text: “T-79.4201 Search Problems and Algorithms 15.5.2006”
- The total number of answer sheets you are submitting for grading

1. Draw the search space corresponding to the 2-SAT formula

$$(x_1 \vee x_2) \wedge (x_2 \vee \bar{x}_3) \wedge (\bar{x}_2 \vee x_3) \wedge (x_1 \vee \bar{x}_2) \wedge (\bar{x}_1 \vee x_2) \wedge (\bar{x}_1 \vee x_3)$$

as a cube, and mark down at the corners of this cube the values of the objective function indicating the number of unsatisfied clauses at each point (= truth assignment). Trace the progress of a (a) GSAT, (b) WalkSAT search with $p = 0$ along the corners of the cube, starting at initial point $(x_1, x_2, x_3) = (1, 0, 0)$.

2. a) Give a solution to the following constraint satisfaction problem (CSP)

$$\langle C_1(y,z), C_2(x,z); x \in \{1,2,3\}, y \in \{1,2,3\}, z \in \{1,2,3\}\rangle$$

where $C_1 = \{(1,3),(3,1),(2,2)\}$, and
$C_2 = \{(1,1),(1,2),(2,1),(3,1),(3,2)\}$.
b) Explain when a constraint satisfaction problem is hyper-arc consistent and study whether the CSP above is hyper-arc consistent.
c) Give a Boolean circuit that computes the Boolean function $f(x_1, x_2, x_3)$ in the table on the right.

x_1	x_2	x_3	f
0	0	0	0
0	1	0	1
1	0	0	1
1	1	0	1
0	0	1	1
0	1	1	1
1	0	1	0
1	1	1	1

3. a) Give the following linear program in the standard form:

$$\begin{aligned}&\max\ x_1 - x_2 - x_3 \quad \text{s.t.}\\ &x_1 + x_2 \geq -2x_3\\ &x_2 - 4x_3 \leq x_1\\ &x_1 \geq 0\\ &x_2 \geq 0\end{aligned}$$

b) Express the condition “$x_0 = 1$ if $x_1 = 1$ or $x_2 = 1$ else $x_0 = 0$” as a set of linear constraints, where for $i = 0,1,2$, x_i is an integer variable such that $0 \leq x_i \leq 1$.

c) Express the condition “if $y = 0$, then $x_1 \geq 128 - x_2$” as a set of linear constraints, where y is an integer variable such that $0 \leq y \leq 1$ and $x_1, x_2 \geq 0$.

4. Suppose you were designing a Genetic Algorithm for solving the following NP-complete NUMBER SET BIPARTITION problem:

INSTANCE: A set of $2n$ natural numbers $A = \{a_1, \ldots, a_{2n}\} \subseteq \mathbb{N}$.
QUESTION: Is there a subset $B = \{b_1, \ldots, b_n\} \subseteq A$ containing exactly half the numbers and sum:

$$\sum_{i=1}^{n} b_i = \frac{1}{2} \sum_{j=1}^{2n} a_j \quad ?$$

Formulate this task as an optimisation problem by defining an appropriate objective function. What would you choose as the individuals (“chromosomes”) in your algorithm? How would you perform recombination (“crossover”) and mutation of the individuals?

Points: 12+12+13+13 = 50p.

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