T-79.4201 **Autumn 2006**
Search Problems and Algorithms
Tutorial 2, 28 September
Problems

1. Study under what conditions an n-wheel graph has a 3-coloring where an n-wheel graph is a pair (V, E) such that $V = \{0, 1, \ldots, n\}$ and $E = \{(0, i) \mid 1 \leq i \leq n\} \cup \{(i, i+1) \mid 1 \leq i < n\} \cup \{(n, 1)\}$.

2. Show that the VERTEX COVER problem can be reduced to the CLIQUE problem where

   VERTEX COVER
   INSTANCE: A graph $G = (V, E)$ and an integer B.
   QUESTION: Is there a set $C \subseteq V$ with $|C| \leq B$ such that if $(v, u) \in E$, then $v \in C$ or $u \in C$.

3. Show that if the SAT decision problem can be solved in polynomial time so can the SAT search problem by writing an algorithm that solves the search problem using an algorithm for the decision problem as a subroutine.

4. Show that if the MAX CUT decision problem can be solved in polynomial time so can the MAX CUT optimization problem by writing an algorithm that solves the optimization problem using an algorithm for the decision problem as a subroutine.

   MAX CUT
   INSTANCE: A graph $G = (V, E)$, a function c giving each edge $e \in E$ an integer capacity $c(e)$ and an integer B.
   QUESTION:
   (D) Does the graph have a cut of size at least B?
   (O) Find a cut with the maximum size.

   A cut in a graph $G = (V, E)$ is a partition of the vertices into two nonempty sets S and $V - S$ and the size of a cut $(S, V - S)$ is the sum of the capacities of the edges between S and $V - S$.