Helsinki University of Technology
Laboratory for Theoretical Computer Science
Pekka Orponen (tel. 5246), Ilkka Niemelä (tel. 3290)

T-79.4201 Search Problems and Algorithms (4 cr)
Exam Thu 21 Dec 2006, 9–12 a.m.

Write down on each answer sheet:
- Your name, department, and student number
- The text: “T-79.4201 Search Problems and Algorithms 21.12.2006”
- The total number of answer sheets you are submitting for grading

1. Consider the following NP-complete graph MIN BISECTION problem:

   INSTANCE: A graph $G = (V,E)$ with an even number $2n$ of vertices.
   QUESTION: Find a minimum width (capacity) balanced cut of G, i.e. a partitioning of the vertices of G into two equal-size sets S and $V-S$, $|S| = |V-S| = n$, such that the number of edges crossing the cut from S to $V-S$ is minimised.

   Describe some simple local search heuristic to search for good bisections of a given input graph G. Draw the complete search space (states, neighbourhood structure, objective function values) for your algorithm in the case of an input graph which has four vertices $\{a,b,c,d\}$, of which each pair except $\{b,d\}$ are connected by an edge. How many states would the search space have in the case of an input graph with six vertices? What about the number of neighbours per state?

2. Consider constraints

   $$\begin{aligned} &\text{alldiff} = \{(1,2,3),(1,3,2),(2,1,3),(2,3,1),(3,1,2),(3,2,1)\} \\ &\text{noteq} = \{(1,2),(1,3),(2,1),(2,3),(3,1),(3,2)\} \end{aligned}$$

   a) Explain briefly what it means that two constraint satisfaction problems (CSPs) are equivalent and then for a CSP C_1

   $$\langle \text{alldiff}(x_1,x_2,x_3); x_1 \in D_1, x_2 \in D_2, x_3 \in D_3 \rangle$$

   give a CSP C_2 in which only the constraint noteq is used and which is equivalent with C_1 when $D_1 = D_2 = D_3 = \{1,2,3\}$.

   b) Explain when a constraint satisfaction problem is hyper-arc consistent and study whether the CSPs C_1 (above) and C_2 (developed by you above) are hyper-arc consistent when $D_1 = \{1,2,3\}$ and $D_2 = D_3 = \{1,2\}$.

3. a) Express the condition “the absolute value of variable x is at least 3” ($|x| \geq 3$) as a set of linear constraints when $-1000 \leq x \leq 1000$.

   b) Consider the following integer programming problem

   $$\begin{aligned} &\max\ -x_1 - 3x_2 \text{ s.t.} \\ &2x_1 - 5x_2 \leq 16 \\ &-6x_1 + x_2 \geq -3 \\ &x_1 \geq 0 \\ &x_1, x_2 \text{ are integers} \end{aligned}$$

   Give the linear relaxation of the problem and transform it to the Simplex tableau form.

   c) Give a basic feasible solution for the linear relaxation above in the Simplex tableau form.

   d) Is the solution that you gave in item 3c) optimal? Justify your answer.

4. Design a Branch and Bound method for solving the MIN BISECTION optimisation problem discussed in Problem 1. Indicate in particular what is your notion of a partial solution, and what lower bounding heuristic you are using to prune the search. Present a small example of how your method works.

Grading: Each problem 10p, total 40p.

KÄÄNNÄ