T-79.3001 Logic in computer science: foundations **Spring 2008**
Exercise 2 ([NS 1997], Chapter I, Sections 2 and 3)
February 6–8, 2008

Tutorial problems

1. Give definitions for the connectives in propositional logic using implication $(\rightarrow)$ and negation $(\neg)$.

2. a) Let $\mathcal{A} = \{A, C\}$ be a truth assignment. Find the truth value of

$$C \wedge (A \leftrightarrow B) \rightarrow ((A \vee \neg B) \wedge (B \vee \neg A) \rightarrow C)$$

by using (i) truth tables and (ii) the definition of truth values. What can be said about the validity, satisfiability, and unsatisfiability of the proposition?

b) Apply truth tables to see whether $\{A \rightarrow B, B \rightarrow C\} \models A \rightarrow C$ holds.

3. Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be truth assignments such that $\mathcal{A}_1 \subseteq \mathcal{A}_2$. A propositional statement ϕ is *positive*, if it consists only of atomic propositions, conjunctions $(\wedge)$ and disjunctions $(\vee)$.

Prove by induction that for all positive propositional statements ϕ, if $\mathcal{A}_1 \models \phi$, then $\mathcal{A}_2 \models \phi$. Explain why this does not hold for all propositional statements.

Demonstration problems

4. Let $\mathcal{A} = \emptyset$ be a truth assignment. Find the truth value of

$$(\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B)$$

by using

a) the truth table and

b) the definition of truth values.

5. Give definitions for the connectives in propositional logic using

a) the proposition that is always false $(\perp)$ and implication $(\rightarrow)$, and

b) the Sheffer stroke.

6. List all possible binary connectives (16 in total) and give their definitions using the basic connectives in propositional logic.

7. Define the Sheffer stroke using the Peirce arrow.

8. An engineer designed a specification for two traffic light posts positioned in the intersection of two one-way streets:

(i) Both the light posts have a green, a yellow and a red light. Exactly one of the lights in each light post is lit at all times.

(ii) Both green lights are not lit at the same time.

(iii) If one lamp post has the red light on, then the other has either the green or the yellow light on.

a) Formalize the above requirements as a set of propositional statements.

b) Construct a truth table for the set of statements.

c) Give (i) a model for the set of statements, and (ii) a truth assignment such that the set of statements is not satisfied.

d) Are the requirements complete enough for a real life situation?

9. Apply truth tables to see whether the following claims hold.

a) $(A \to B) \to ((B \to C) \to (A \to C))$ is valid.

b) $\neg((A \to B) \to ((\neg A \to B) \to B))$ is unsatisfiable.

c) $A \leftrightarrow B$ and $\neg(A \leftrightarrow \neg B)$ are logically equivalent.

d) $\{(A \wedge B) \vee (C \wedge A), (A \wedge B) \vee \neg B\} \models A \vee (C \wedge \neg B)$.

10. Let $\mathcal{A}_1 \subseteq \mathcal{P}$ and $\mathcal{A}_2 \subseteq \mathcal{P}$ be truth assignments and $\phi \in \mathcal{L}$ a proposition. Show that if $\mathcal{A}_1 \cap At(\phi) = \mathcal{A}_2 \cap At(\phi)$, then $\mathcal{A}_1 \models \phi \iff \mathcal{A}_2 \models \phi$.