T-79.3001 Logic in computer science: foundations **Spring 2008**
Exercise 12 ([Huth & Ryan, 2000], Chapter 4)
Apr 29–30 and May 2, 2008

Tutorial problems

1. (a) Write down a program P such that $\models_p [\texttt{true}]\ P\ [\texttt{y==x+2}]$ holds and prove that this is so.

(b) Write down a program P such that P contains an if-statement and

$$\models_p [\texttt{true}]\ P\ [\texttt{z>x+y+4}]$$

holds, and prove that this is so.

2. Show that the following holds for program `Prog`:

$$\models_p [\texttt{true}]\ \texttt{Prog}\ [\texttt{x==v-z}],$$

where `Prog` is as follows:

```
x=0;
y=0;
while(!(y ==z)){
    y=y+1;
    x=x-1
}
x=x+v;
```

3. (a) For any B_1, B_2 and P explain why $\models_p [B_1]\ P\ [B_2]$, whenever the relation $\models_t [B_1]\ P\ [B_2]$ holds.

(b) Show that the following holds for program `Prog` in Exercise 2:

$$\models_t [\texttt{z>=0}]\ \texttt{Prog}\ [\texttt{x==v-z}].$$

Demonstration problems

4. Use propositional logic to prove the equivalence of the following statements.

(a) `!(a==b||a<b)`

(b) `a!=b&&!(b>a)`

5. Prove the partial correctness in the following cases.

(a) $\models_p$ [x>0] y=x+1 [y>1]

(b) $\models_p$ [true] y=x ; y=x+x+y [y==3*x]

(c) $\models_p$ [x>1] a=1 ; y=x ; y=y - a [y>0&&x>y]

6. Show that $\models_p$ [true] P [z== min (x, y)], where P is the following program:

```
if(x>y) then {
    z=y
} else {
    z=x
}
```

7. Show that

(a) $\models_p$ [true] Sum [z==x+y]

(b) $\models_t$ [0<=y] Sum [z==x+y]

where Sum is the following program:

```
z=x;
v=y;
while(!(v ==0)) {
    z=z+1;
    v=v-1
}
```