T-79.3001 Logic in Computer Science: Foundations

Examination, January 8, 2009

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)

(a) Define the following concepts: *an adequate set of connectives*, *Herbran universe*, and *total correctness*. ($3 \times 2p$)

(b) What is meant by the notation $\Sigma \models \phi$?

Prove in detail that if $\Sigma \cup \{\phi\} \models \psi$, then $\Sigma \models \phi \rightarrow \psi$. (4$p$)

Assignment 2 (10p) Prove the following claims using semantic tableaux:

(a) $\models (A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$

(b) $\models \exists x(P(x) \vee Q(x) \rightarrow \forall x(P(x) \vee Q(x)))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg(\forall x \exists y(P(x) \wedge Q(y)) \rightarrow \exists y \forall x(P(x) \wedge Q(y))).$$

Make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 (10p) Let us represent strings “”, “a”, “b”, “aa”, “ab”, “ba”, “bb”, ... that consist of letters a ja b using ground terms

$$e,\ a(e),\ b(e),\ a(a(e)),\ a(b(e)),\ b(a(e)),\ b(b(e)),\ \ldots,$$

built of a constant symbol e, which represents the empty string “”, and unary functions $a(x)$ and $b(x)$, that append the respective letter a or b at the beginning of a string x. Thus $a(b(e))$ is interpreted as $a(b(“”)) = a(“b”) = “ab”$.

(a) Define predicate $AB(x) =$ “the string x is of the form $abab\ldots ab$ where the string ab repeats $n \geq 0$ times” using predicate logic so that your definition covers all finite strings represented as explained above.

(b) Give a model $\mathcal{S} \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \not\models AB(b(a(e))).$$

Assignment 5 (10p)

Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if}(B)\ \texttt{then}\ \{C_1\}\ \texttt{else}\ \{C_2\}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Minus`:

```
v=x;z=y;while(!(z==0)){z=z-1;v=v-1}.
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Minus}\ [\texttt{v==x-y}].$$

The name of the course, the course code, the date, your name, your student number, and your signature must appear on every sheet of your answers.