

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)

- (a) Define the following concepts: *ground term*, *modus ponens*, and the *scope* of a quantifier. (3 × 2p)
- (b) What is meant by the notation $Cn(\Sigma)$?
Prove in detail that if $\Sigma_1 \subseteq \Sigma_2$, then $Cn(\Sigma_1) \subseteq Cn(\Sigma_2)$. (4p)

Assignment 2 (10p) Prove the following claims using semantic tableaux:

- (a) $\models (A \rightarrow B) \wedge (B \rightarrow C) \wedge (C \rightarrow A) \rightarrow (A \leftrightarrow C)$
- (b) $\models \forall x \exists y (P(x) \wedge Q(y)) \rightarrow \exists y \forall x (P(x) \wedge Q(y))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg \exists x \forall y (\forall z R(x, z) \rightarrow \forall x R(x, y)).$$

Make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 (10p) Let us represent natural numbers $0, 1, 2, \dots$ using ground terms $0, s(0), s(s(0)), \dots$ built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x .

- (a) Define a predicate $D(x, y, z) =$ “the distance between numbers x and y is z ” using sentences of predicate logic so that your definition covers all natural numbers (represented in the way explained above).
- (b) Give a model $\mathcal{S} \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \not\models \exists x \exists y (D(x, x, x) \wedge D(y, y, y) \wedge \neg(x = y)).$$

Assignment 5 (10p)

Explain how the *weakest precondition* B_1 of an if-statement

$$\text{if}(B) \text{ then } \{C_1\} \text{ else } \{C_2\}$$

can be formed given a postcondition B_2 for it.

Consider the following program Minus:

$$v = x ; z = y ; \text{while}(! (z == 0)) \{ z = z - 1 ; v = v - 1 \}.$$

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\text{true}] \text{Minus} [v == x - y].$$

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.