T-79.3001 Logic in computer science: foundations **Spring 2007**
Exercise 7 ([NS, 1997], Predicate Logic, Chapters 1–4)
March 20–22, 2007

The second periodic time tracking questionnaire is open 16th–23th March at `http://www.cs.hut.fi/cgi-bin/teekysely.pl?action=showform&id=T793001-T-79.3001_2007ajankaytto2`

If you answer all the questionnaires in time, you get two bonus points for the exam, see `http://www.tcs.hut.fi/Studies/T-79.3001/2007SPR/index.shtml#feedback` for more details.

Tutorial problems

1. Formalize the following statements in predicate logic:

a) If all birds fly, a penguin is not a bird.
b) The children of two siblings are cousins.
c) There is only one real Santa Claus.

Draw the syntax trees for the sentences.

2. Formalize the following statements in predicate logic:

> "Penguins are black and white. Some old tv shows are black and white. Therefore, some penguins are old tv shows."

Give a structure that doesn't satisfy your formalization.

3. A graph is a set S of nodes and a set E of edges between the nodes ($E \subseteq S \times S$). Set of nodes $P \subseteq S$ is a *vertex cover* for the graph, if for all $\langle s, s' \rangle \in E$ it holds $s \in P$ or $s' \in P$. The problem of *vertex covering* is to find a vertex cover for a graph.

a) Formalize the vertex covering problem using predicate logic.
b) Give a model for your formalization.
c) Give a structure that doesn't satisfy your formalization.

Demonstration problems

4. Formalize the following sentences using predicate logic:

a) There is a faulty gate.

b) This algorithm is the fastest.

c) Each participant of this course has a workstation to use

d) Only one process can write in each file at a time

Draw the syntax trees for sentences a) and b).

5. Remove unnecessary parenthesis so that the meaning of statement does not change.

a) $(\forall y((\exists x(P(x) \wedge Q(x))) \rightarrow L(y)))$

b) $((\exists x(\exists y(P(x,y) \vee Q(y,x)))) \leftrightarrow (\forall x(\neg K(f(x)))))$

c) $(\forall x(\forall y(A \wedge B)))$

6. What ground (variable-free) terms can you compose from a constant c, a unary function symbol f and a binary function symbol g?

7. Represent arbitrary trees with function symbols using at most three constant or function symbols.

8. Show that if $\forall x \phi(x)$ is a sentence and t is a ground term, then $\phi(t)$ is a sentence.

9. Consider a domain $\mathbb{N}^2 = \{\langle x,y \rangle | x \in \mathbb{N}, y \in \mathbb{N}\}$. Choose interpretations for a constant c and a unary function symbol $f \in \mathcal{F}_1$ such that each element in the domain has an interpretation.

10. A graph is a set S of nodes and a set K of edges between the nodes ($K \subseteq S \times S$). The nodes s and s' of the graph are adjacent, if they are connected with an edge ($\langle s, s' \rangle \in K$). Let C be a set of colors. The problem of *node coloring* is to find a color in C for each node of the graph so that each node has a unique color and two adjacent nodes have different colors.

a) Formalize the node coloring problem using predicate logic.

b) Give a model for your formalization.

c) Give a structure that doesn't satisfy your formalization.