

**Tutorial problems**

1. Prove by induction that  $4^n - 1$  is divisible by three for all  $n \geq 0$ . (Hint: a number is divisible by three if it's of the form  $3m$  for some integer  $m$ .)
2. Formalize the following statements in propositional logic:
  - a) I'll have coffee or tea and a sandwich.
  - b) If it rains or the wind is too heavy, we won't go out.
  - c) Either John or Mary will pick up Lisa from day care in the evening.
  - d) There's no smoke without fire.
  - e) When it isn't raining I walk to work, otherwise I drive my car.
3. Remove unnecessary parenthesis from the following propositional statements. What are the forms of the statements? Give parse trees for the propositions.
  - a)  $((A \wedge (\neg B)) \leftrightarrow C) \vee (A \rightarrow (\neg C))$
  - b)  $(\neg(((\neg B) \vee (\neg D)) \rightarrow (B \vee (C \vee (\neg A))))))$
  - c)  $(A \leftrightarrow (D \vee ((B \rightarrow (\neg D)) \wedge C)))$

**Demonstration problems**

4. Prove by induction that a set of  $n$  elements has  $2^n$  subsets.
5. Prove the following claims (sets  $A, B$  and  $C$  are subsets of universe  $E$ ):
  - a)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ .
  - b)  $E - (A \cap B) = (E - A) \cup (E - B)$ .
6. Formalize the following statements in propositional logic:
  - a) I can't finish my work unless you help me.
  - b) I either walk, ride a bicycle, or sometimes drive a car to work.
  - c) Merja and Arto are coming to visit us.
  - d) You won't get dessert because you have been naughty

- e) Even though the manual was long I finished reading it too early.
- f) If somebody asks me — or even if no one does — he shouldn't buy a car or he must live far from his workplace and gasoline should become cheaper.
- 7.** Let  $\mathcal{P} = \{A, B, C\}$  be the set of atomic propositions. Which of the following are propositional statements? Why?
- $A$
  - $\neg(A \wedge B)$
  - $(A \wedge (B \rightarrow (A \wedge C)))$
  - It is raining today.
- 8.** Prove that all propositional statements have an even number of parenthesis.
- 9.** Remove unnecessary parenthesis so that the meaning of the proposition does not change.
- $(A \rightarrow ((B \wedge C) \vee D))$
  - $((A \rightarrow B) \wedge (B \rightarrow C)) \rightarrow (A \rightarrow C)$
  - $((A \wedge (B \vee C)) \vee (A \wedge (C \vee D)))$
  - $((\neg(A \wedge B)) \leftrightarrow ((B \rightarrow C) \wedge A))$
  - $((\neg A) \wedge (\neg B)) \rightarrow \neg(A \vee B)$
- 10.** What are the forms of the propositional statements in the previous exercise? Give parse trees for the propositions.
- 11.** List the substatements of the following propositional statement.

$$(\neg A \rightarrow (\neg B \rightarrow C)) \rightarrow (\neg(\neg A \rightarrow B) \rightarrow C)$$