T-79.3001 Logic in Computer Science: Foundations

Examination, October 31, 2007

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)

(a) Define the following concepts: *formation tree*, *free variable occurrence*, and *unique names assumption*. ($3 \times 2p$)

(b) What is meant by the notation $\phi \equiv \psi$?

Prove in detail that if $\models \phi \to \psi$ and $\models \neg\phi \to \neg\psi$, then $\phi \equiv \psi$.

Assignment 2 (10p) Prove the following claims using semantic tableaux:

(a) $\not\models (\neg A \leftrightarrow B \vee C) \leftrightarrow (A \leftrightarrow \neg B \vee \neg C)$

(b) $\models \exists x \forall y Q(x,y) \to \forall y \exists x Q(x,y)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg \forall x \exists y (\exists z R(x,z) \to \exists v R(y,v)).$$

Try to make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 (10p) Let us represent any finite string consisting of letters a, b, and c using unary function symbols a, b, and c and a constant symbol e denoting the empty string. Thus, for instance, the term $a(x)$ denotes a string that starts with an a followed by a string x, and the ground term $b(a(b(a(e))))$ represents "*baba*".

(a) Define the predicate $L(x,y)$ = "string x strictly precedes string y in the lexicographic order" so that your definition covers all finite strings represented as described above.

(b) Give a model $\mathcal{S} \models \Sigma$ for your definition Σ according to which it holds that

$$\Sigma \not\models \exists x \exists y (L(x,y) \wedge L(y,x)).$$

Assignment 5 (10p)

Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if}(B)\,\texttt{then}\,\{C_1\}\,\texttt{else}\,\{C_2\}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Divide`:

```
v=0;z=x;while(z>=y){z=z-y;v=v+1}.
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Divide}\ [\texttt{v==x/y}],$$

where `x/y` denotes the integer quotient when `x` is divided by `y`.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.