T-79.194 Tietojenkäsittelyteorian seminaari
Linear equalities over reals (cont'd)
Linear inequalities over reals
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Exercises

Name ______________________________________
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1. Consider the following set of linear inequalities:

$$-y \leq 0 \tag{1}$$
$$-y - z + 2 \leq x \tag{2}$$
$$0 \leq x \tag{3}$$
$$-y - 2 \leq x \tag{4}$$
$$y - 3 \leq x \tag{5}$$
$$x \leq -2y + 6 \tag{6}$$

Apply once the x-*ELIMINATION* rule (to all possible inequalities in this set). Write down the resulting set of inequalities.

Answer

Using the x-*ELIMINATION* rule we obtain the following set of inequalities in which x does not appear:

$$-y \leq 0 \tag{7}$$

$$-y - z + 2 \leq -2y + 6 \tag{8}$$

$$0 \leq -2y + 6 \tag{9}$$

$$-y - 2 \leq -2y + 6 \tag{10}$$

$$y - 3 \leq -2y + 6 \tag{11}$$

2. Consider the following set of linear inequalities:

$$0 \leq y \tag{12}$$
$$-x - y + 2 \leq z \tag{13}$$
$$0 \leq x \tag{14}$$
$$-x - y \leq 2 \tag{15}$$
$$-x + y \leq 3 \tag{16}$$
$$x + 2y \leq 6 \tag{17}$$

Examine by using Fourier-Motzkin Elimination, whether this set is consistent or not. Write down the details.

Answer

Trasforming each of them to the x-normal form yields the following set:

$$-y \leq 0 \tag{18}$$
$$-y - z + 2 \leq x \tag{19}$$
$$0 \leq x \tag{20}$$
$$-y - 2 \leq x \tag{21}$$
$$y - 3 \leq x \tag{22}$$
$$x \leq -2y + 6 \tag{23}$$

Using the x-*ELIMINATION* rule we obtain the following set of inequalities in which x does not appear:

$$
\begin{aligned}
-y &\leq 0 & (24)\\
-y - z + 2 &\leq -2y + 6 & (25)\\
0 &\leq -2y + 6 & (26)\\
-y - 2 &\leq -2y + 6 & (27)\\
y - 3 &\leq -2y + 6 & (28)
\end{aligned}
$$

Transforming each of the five inequalities to the y-normal form we now obtain the following set:

$$0 \leq y \tag{29}$$
$$y \leq z + 4 \tag{30}$$
$$y \leq 3 \tag{31}$$
$$y \leq 8 \tag{32}$$
$$y \leq 3 \tag{33}$$

Eliminating now y using the y-*ELIMINATION* rule we obtain the following set of four inequalities:

$$0 \leq z + 4 \tag{34}$$
$$0 \leq 3 \tag{35}$$
$$0 \leq 8 \tag{36}$$
$$0 \leq 3 \tag{37}$$

We can now delete the last three inequalities using the *DELETION* rule and we end up with a single inequality the z-normal form which is:

$$-4 \leq z$$

At this moment we apply the z-*ELIMINATION* rule. We end up with the empty set. This implies that the original set of inequalities is consistent.