T-79.149 Discrete Structures, Autumn 2004

Tutorial 6, 27 October

1. It has been previously established that the egf for the class of derangements is $\hat{d}(z) = e^{-z}/(1-z)$. Derive from this a simple recurrence equation for the number of derangements of n elements. Can you think of a combinatorial interpretation for this formula?

2. Let $h(z) = \sum_{n \geq m} h_n z^n$, where $h_m \neq 0$, be a formal Laurent series. Prove the following results:

   (a) $\mathrm{Res}(h'(z)) = 0$;

   (b) $\mathrm{Res}(h'(z)/h(z)) = m$.

3. Derive from Lagrange's inversion formula for formal power series (Theorem 5.2 in the lecture notes) its following reformulation (useful e.g. in the analysis of tree structures): Let $f(z)$ and $\phi(u)$ be formal power series satisfying $\phi(0) = \phi_0 \neq 0$ and $f(z) = z\phi(f(z))$. Then for all $n \geq 1$:

$$[z^n]f(z) = \frac{1}{n}[u^{n-1}]\phi(u)^n.$$

   (*Hint:* Consider the power series $\psi(u) = \frac{u}{\phi(u)}$.)

4. Derive formulas for the number of n-node rooted ordered trees and n-node binary trees (rooted ordered trees where each node has 0, 1 or 2 descendants) directly by applying the respective ogf-constructions and Lagrange's inversion formula.