T-79.149 Discrete Structures, Autumn 2004

Tutorial 2, 29 September

1. Let $F(X) = \sum_{n\geq 0} a_n X^n$ be a formal power series. Prove the following results:

   (a) $F' = 0$ if and only if $F = a_0 =$ constant;

   (b) $F' = F$ if and only if $F = a_0 \cdot \mathrm{Exp}(X)$.

2. Let $F(X) = \sum_{n\geq 0} a_n X^n$, $G(X) = \sum_{m\geq 0} b_m X^m$ and $F_i(X) = \sum_{k\geq 0} c_{ik} X^k$ $(i = 0, 1, \ldots)$ be formal power series. Prove the following rules concerning the differentiation of products, infinite sums, and compositions of series:

$$\begin{aligned} D\, F(X)G(X) &= F'(X)G(X) + F(X)G'(X), \\ D \sum_{i\geq 0} F_i(X) &= \sum_{i\geq 0} F_i'(X), \\ D\, G(F(X)) &= G'(F(X)) \cdot F'(X). \end{aligned}$$

   Which constraints on the coefficient sequences does one need to take into account when applying these rules?

3. Let $F(X) = \sum_{n\geq 1} a_n X^n$ and $G(X) = \sum_{m\geq 1} b_m X^m$ be formal power series that satisfy $a_0 = b_0 = 0$ ja $a_1, b_1 \neq 0$. Show that if $F(G(X)) = X$, then also $G(F(X)) = X$. (Thus for a given series F the "right" and "left" inverse series $G = F^{[-1]}$ coincide.) Determine by a formal calculation the three first coefficients of the series $\mathrm{Ln}(1 + X) = (\mathrm{Exp}(X) - 1)^{[-1]}$.