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T-79.148 Introduction to Theoretical Computer Science (2 cr)
Exam Tue 17 Dec 2002, 9–12 a.m.

Write down on each answer sheet:
- Your name, department, and study book number
- The text: "T-79.148 Introduction to Theoretical Computer Science 17.12.2002"

In case you are taking the exam as an "old" student, i.e. you have participated in the course already in Spring 2002 or earlier, write also visibly on each answer sheet the text "OLD" and the semester in which you attended the course (e.g. "OLD SPRING 2002"). In this case your exam will be graded according to the rules of Spring 2002, otherwise according to the rules of Autumn 2002. According to the new rules, you must e.g. complete all your Regis computerised assignments before taking the exam; also the way of calculating various bonus exam points differs from earlier installments. You can only take the exam as an "old" student if you have really attended the course before the present semester.

1. (a) Give a regular expression that describes the language

$$\{w \in \{0,1\}^* \mid w \text{ contains 0110 or 1001 as a substring (possibly both)}\}.$$

8p.

   (b) Design a deterministic finite automaton that recognises the language in part (a). *7p.*

2. (a) Design context-free grammars for the languages $L_{\leq} = \{a^i b^j \mid 0 \leq i \leq j\}$ and $L_{\neq} = \{a^i b^j \mid i \neq j\}$. (*Hint:* Note that $i \neq j$ if and only if $i < j$ or $i > j$.) *6p.*

   (b) Prove (precisely!) that the language $L_{\leq}$ in part (a) is not regular. *6p.*

   (c) Prove (precisely!) that the language $L_{\neq}$ in part (a) is not regular. *3p.*

3. Design a deterministic pushdown automaton that recognises (accepts) the language $L_{\leq}$ considered in problem 2. (Present the automaton preferably as a state diagram rather than a transition table.) Show the computation sequences of your automaton on the inputs abb, bb and aba. *15p.*

4. *One* of the following:

   (a) Show that if the language $L \subseteq \{0,1\}^*$ is regular, then so are the following languages, consisting of all the prefixes and suffixes of the words in L:

$$\begin{aligned} \mathrm{Pref}_L &= \{x \in \{0,1\}^* \mid xy \in L \text{ for some } y \in \{0,1\}^*\}, \\ \mathrm{Suff}_L &= \{y \in \{0,1\}^* \mid xy \in L \text{ for some } x \in \{0,1\}^*\}. \end{aligned}$$

15p.

   (b) Formulate and state precisely so called "Rice's theorem", and apply it in some example case. (You do not need to prove the theorem, but you must define precisely all the concepts needed for its statement. Also in the application example you must indicate exactly *how* the theorem is being applied.) *15p.*

Total 60p.