T-79.148 Spring 2005

Introduction to Theoretical Computer Science Tutorial 8, 15–18 March Problems

## Homework problems:

1. Convert the following grammar into Chomsky normal form:

$$S \rightarrow AB \mid c$$

$$A \rightarrow T \mid aA$$

$$B \rightarrow TT \mid \varepsilon$$

$$T \rightarrow bS$$

2. Determine, using the CYK algorithm ("dynamic programming method", Sipser p. 241, Lewis & Papadimitriou p. 155), whether the strings *abba*, *bbaa* and *bbaab* are generated by the grammar

$$\begin{array}{ccc} S & \rightarrow & AB \mid BA \mid a \mid b \\ A & \rightarrow & BA \mid a \\ B & \rightarrow & AB \mid b \end{array}$$

In the positive cases, give also the respective parse trees.

3. Design pushdown automata recognising the following languages:

(a) 
$$\{w \in \{a,b\}^* \mid w \text{ has as many } a\text{'s as } b\text{'s}\}$$
  
(b)  $\{w \in \{a,b\}^* \mid w=w^R\}$ 

- 4. Design an algorithm for testing whether a given a context-free grammar  $G = (V, \Sigma, P, S)$ , generates a nonempty language, i.e. whether any terminal string  $x \in \Sigma^*$  can be derived from the start symbol S.
- 5. Design a pushdown automaton corresponding to the grammar  $G=(V,\Sigma,P,S),$  where

$$\begin{split} V &= \{S, (,), ^*, \cup, \emptyset, a, b\} \\ \Sigma &= \{(,), ^*, \cup, \emptyset, a, b\} \\ P &= \{S \rightarrow (SS), S \rightarrow S^*, S \rightarrow (S \cup S), \\ S \rightarrow \emptyset, S \rightarrow a, S \rightarrow b\} \end{split}$$

6. Design a grammar corresponding to the pushdown automaton  $M=(Q,\Sigma,\Gamma,\Delta,s,F),$  where

$$\begin{split} Q = & \{s,q,f\}, \ \Sigma = \{a,b\}, \ \Gamma = \{a,b,c\}, \ F = \{f\}, \\ \Delta = & \{\left((s,e,e),(q,c)\right), \left((q,a,c),(q,ac)\right), \left((q,a,a),(q,aa)\right) \\ & \left((q,a,b),(q,e)\right), \left((q,b,c),(q,bc)\right), \left((q,b,b),(q,bb)\right) \\ & \left((q,b,a),(q,e)\right), \left((q,e,c),(f,e)\right) \} \end{split}$$