T-79.148 **Spring 2004**
Introduction to Theoretical Computer Science
Tutorial 7
Problems

Homework problems:

1. Design a right-linear grammar that generates the language

$$\{w \in \{0,1\}^* \mid \text{ the number of 1s in } w \text{ is divisible by 3}\}$$

(Cf. Problem 3/1d.)

2. Desing a context-free grammar that produces all syntactically correct regular expressions over the alphabet $\Sigma = \{a, b\}$. Give a parse tree for the expression $(a \cup bb)^*a$.

3. Construct a context-free grammar for the language:

$$\{a^i b^j a^k \mid i \geq j \text{ or } i \geq k\}$$

Is your grammar ambiguous?

Demonstration problems:

4. Prove that the class of context-free languages is closed under unions, concatenations, and the Kleene star operation, i.e. if the languages $L_1, L_2 \subseteq \Sigma^*$ are context-free, then so are the languages $L_1 \cup L_2$, $L_1 L_2$ and L_1^*.

5. (a) Prove that the following context-free grammar is ambiguous:

$$\begin{aligned} S &\rightarrow \textbf{if } b \textbf{ then } S \\ S &\rightarrow \textbf{if } b \textbf{ then } S \textbf{ else } S \\ S &\rightarrow s. \end{aligned}$$

   (b) Design an unambiguous grammar that is equivalent to the grammar in item (a), i.e. that generates the same language. (*Hint:* Introduce new nonterminals B and U that generate, respectively, only “balanced” and “unbalanced” **if-then-else-**sequences.)

6. Design a recursive-descent (top-down) parser for the grammar from Problem 6/6.