T-79.148 **Spring 2004**
Introduction to Theoretical Computer Science
Tutorial 1
Problems

Homework problems:

1. Let $A = \{a, b, c\}$, $B = \{a, e\}$ ja $C = \{a, c, d\}$. Write out the following three sets:
   (a) $A \cup (C - B)$;
   (b) $B \times (A \cap C)$;
   (c) $\mathcal{P}(\{\emptyset\}) - \mathcal{P}(\emptyset)$.

2. Let $A = \{a, b, c, d\}$, and define a relation $R \subseteq A \times A$ as follows:
$$R = \{\{(a, d), (b, b), (b, c), (c, a), (d, c)\}$$
Draw the graphs corresponding to the following relations:
$$\text{(a)}\ R, \qquad \text{(b)}\ R^{-1}, \qquad \text{(c)}\ R \circ R, \qquad \text{(d)}\ (R \circ R) - R.$$
Are some of these relations actually functions?

3. Verify by induction the correctness of the formula:
$$1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n + 1)(2n + 1).$$

Demonstration problems:

4. Define a relation $\sim$ on the set $\mathbb{N} \times \mathbb{N}$ by the rule:
$$(m, n) \sim (p, q) \quad \Leftrightarrow \quad m + n = p + q.$$
Prove that this is an equivalence relation, and describe intuitively ("geometrically") the equivalence classes it determines.

5. Prove by induction that if X is a finite set of cardinality $n = |X|$, then its power set $\mathcal{P}(X)$ is of cardinality $|\mathcal{P}(X)| = 2^n$.

6. Prove by induction that every partial order defined on a finite set S contains at least one minimal element. Furthermore, provide examples showing that the minimal element is not necessarily unique (i.e. there can be more than one), and that in an infinite set S the claim does not necessarily hold.