T-79.148 **Spring 2003**
Introduction to Theoretical Computer Science
Tutorial 1, 20–22 January
Problems

Homework problems:

1. Let $A = \{a, b, c, d\}$, and define a relation $R \subseteq A \times A$ as follows:

$$R = \{(a, b), (a, d), (b, d), (c, b), (c, d), (d, d)\}.$$

Draw the graphs corresponding to the following relations:

$$\text{(a)}\ \ R, \qquad \text{(b)}\ \ R^{-1}, \qquad \text{(c)}\ \ R \circ R, \qquad \text{(d)}\ \ R - (R \circ R).$$

Are some of these relations actually functions?

2. Let A and B be subsets of a given fundamental set U. Prove the correctness of the following *de Morgan formulas* that relate the unions, intersections, and complements of A and B to each other:

$$\overline{A \cup B} = \bar{A} \cap \bar{B}, \quad \overline{A \cap B} = \bar{A} \cup \bar{B}.$$

3. (a) List all the equivalence relations (partitions) on the set $\{a, b, c\}$.
   (b) Draw the Hasse diagrams or graph representations for all the order relations (partial orders) on the set $\{a, b, c\}$.

Demonstration problems:

4. Define a relation $\sim$ on the set $\mathbb{N} \times \mathbb{N}$ by the rule:

$$(m, n) \sim (p, q) \quad \Leftrightarrow \quad m + n = p + q.$$

Prove that this is an equivalence relation, and describe intuitively ("geometrically") the equivalence classes it determines.

5. Prove by induction that if X is a finite set of cardinality $n = |X|$, then its power set $\mathcal{P}(X)$ is of cardinality $|\mathcal{P}(X)| = 2^n$.

6. Prove by induction that every partial order defined on a finite set S contains at least one minimal element. Furthermore, provide examples showing that the minimal element is not necessarily unique (i.e. there can be more than one), and that in an infinite set S the claim does not necessarily hold.