T-79.148 **Autumn 2003**
Introduction to Theoretical Computer Science
Tutorial 3
Problems

Homework problems:

1. Design finite automata that recognise the following languages:
   (a) $\{w \in \{a,b\}^* \mid w \text{ contains } aba \text{ as a substring}\}$;
   (b) $\{w \in \{a,b\}^* \mid w \text{ does not contain } bba \text{ as a substring}\}$;
   (c) $\{w \in \{0,1\}^* \mid \text{the number of 0's in } w \text{ is divisible by three}\}$;
   (d) $\{w \in \{a,b\}^* \mid 11 \text{ occurs exactly twice as a substring in } w\}$;
   (e) $\{w \in \{a,\ldots,z,0,\ldots,9,.,@\}^* \mid w \text{ is a valid e-mail address}\}$;

2. Design a finite automaton that accepts precisely those binary strings where the number of 0's is odd and the number of 1's is divisible by three (e.g. 000111, 1101 and 0, but not 1010 or 111). [NB. In this and similar problems in the future, it is for simplicity always assumed that also zero is even, divisible by three, etc., unless otherwise indicated.]

3. Design a finite automaton that models the behaviour of a lift moving between two storeys. The lift can be either up or down. Both storeys have a simple 'call here' button for the lift, and inside the lift there are buttons for going 'up' and 'down'. In addition, the lift has a door that can be opened and closed; the lift only moves when the door is closed. The time required for the lift to travel between the two storeys does not need to be taken into account, and any possible service requests occurring during this interval can be ignored. The automaton does not need to have any distinct "final states".

Demonstration problems:

4. Formulate the model of a simple coffee machine presented in class (lecture notes p. 17) precisely according to the mathematical definition of a finite automaton (Definition 2.1). What is the formal language recognised by this automaton?

5. Design finite automata that recognise the following languages:
   (a) $\{a^m b^n \mid m = n \mod 3\}$;
   (b) $\{w \in \{a,b\}^* \mid w \text{ contains equally many } a\text{'s and } b\text{'s, modulo } 3\}$.

   (The notation "$m = n \mod 3$" means that the numbers m and n yield the same remainder when divided by three.)

6. Design a finite automaton that recognises sequences of integers separated by plus and minus signs (e.g. 11+20-9, -5+8). Implement your automaton as a computer program that also calculates the numerical value of the input expression.