T-79.148 **Autumn 2002**
Introduction to Theoretical Computer Science
Tutorial 3, 2–4 October
Problems

Homework problems:

1. Design finite automata that recognise the following languages:

   (a) $\{w \in \{a, b\}^* \mid w \text{ contains } ab \text{ as a substring}\}$;
   (b) $\{w \in \{a, b\}^* \mid w \text{ contains } aba \text{ as a substring}\}$;
   (c) $\{w \in \{a, b\}^* \mid w \text{ does not contain } aba \text{ as a substring}\}$;
   (d) $\{w \in \{a, b\}^* \mid w \text{ contains exactly two occurrences of the substrings } ab \text{ and/or } ba\}$;
   (e) $\{w \in \{a, b\}^* \mid w \text{ contains an even number of } a\text{'s}\}$;
   (f) $\{w \in \{a, \ldots, z, 0, \ldots, 9, ., @\}^* \mid w \text{ is a valid e-mail address}\}$;
   (g) $\{w \in \{a, \ldots, z, 0, \ldots, 9, ., @\}^* \mid w$ is a valid e-mail address ending in the country code '.fi' for Finland$\}$.

2. Design a finite automaton that accepts precisely those binary strings that contain an even number of both 0's and 1's (e.g. 0011 and 1010, but not 001). [NB. In this and similar problems in the future, it is for simplicity always assumed that also zero is an even number, unless otherwise indicated.]

3. Design a finite automaton that models the behaviour of a lift moving between two storeys. The lift can be either up or down. Both storeys have a simple 'call here' button for the lift, and inside the lift there are buttons for going 'up' and 'down'. In addition, the lift has a door that can be opened and closed; the lift only moves when the door is closed. The time required for the lift to travel between the two storeys does not need to be taken into account, and any possible service requests occurring during this interval can be ignored. The automaton does not need to have any distinct "final states".

Demonstration problems:

4. Formulate the model of a simple coffee machine presented in class (lecture notes p. 17) precisely according to the mathematical definition of a finite automaton (Definition 2.1). What is the formal language recognised by this automaton?

5. Design finite automata that recognise the following languages:

   (a) $\{a^m b^n \mid m = n \mod 3\}$;
   (b) $\{w \in \{a, b\}^* \mid w \text{ contains equally many } a\text{'s and } b\text{'s, modulo } 3\}$.

   (The notation "$m = n \mod 3$" means that the numbers m and n yield the same remainder when divided by three.)

6. Design a finite automaton that recognises sequences of integers separated by plus and minus signs (e.g. 11+20-9, -5+8). Implement your automaton as a computer program that also calculates the numerical value of the input expression.