**T-79.148** **Autumn 2002**
**Introduction to Theoretical Computer Science**
**Tutorial 1, 18–20 September**
**Problems**

**Homework problems:**

1. Let $A = \{a, b, c\}$, $B = \{b, d\}$, and $C = \{a, c, d, e\}$. List the elements of the following sets:

   (a) $A \cup (C - B)$;

   (b) $B \times (A \cap C)$;

   (c) $\mathcal{P}(\{\emptyset\}) - \mathcal{P}(\emptyset)$.

2. Let $A = \{a, b, c, d\}$, and define a relation $R \subseteq A \times A$ as follows:

$$R = \{(a, b), (a, c), (b, c), (c, c), (d, b)\}.$$

   Draw the graphs corresponding to the following relations:
   a) $R$, c) $R \circ R$,
   b) $R^{-1}$, d) $R \cup (R \circ R)$.
   Are some of these relations actually functions?

3. (a) List all the equivalence relations (partitions) on the set $\{a, b, c\}$.

   (b) Verify by induction the correctness of the formula:

$$1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n + 1)(2n + 1).$$

**Demonstration problems:**

4. Prove the correctness of the distributive laws for set unions and intersections:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C),$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

5. Define a relation $\sim$ on the set $\mathbb{N} \times \mathbb{N}$ by the rule:

$$(m, n) \sim (p, q) \quad \Leftrightarrow \quad m + n = p + q.$$

   Prove that this is an equivalence relation, and describe intuitively ("geometrically") the equivalence classes it determines.

6. Prove by induction that if $X$ is a finite set of cardinality $n = |X|$, then its power set $\mathcal{P}(X)$ is of cardinality $|\mathcal{P}(X)| = 2^n$.