Tik-79.148 **Spring 2001**
Introduction to Theoretical Computer Science
Tutorial 2
Exercises

Ordinary exercises:

1. Prove by induction:

$$1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1),$$

when $n \geq 1$.

2. Let S be a non-empty finite set.

   a) Define an injective function $f : S \to 2^S$.
   b) Show that it is not possible to construct an injective function $g : 2^S \to S$.

3. Let $\Sigma = \{a, b\}$ be an alphabet. Find the words that belong into the following languages:

   a) $L_1 = \{w \mid w \in \Sigma\Sigma\}$
   b) $L_2 = \{axbx \mid x \in \Sigma\}$
   c) $L_3 = L_1 \circ L_2$

Demonstration exercises:

4. Prove by induction: Every partial order on a nonempty finite set S has at least one minimal element. Does the result necessarily hold if S is infinite?

5. Use the pigeonhole principle to show that in any group of at least two people there are at least two persons that have the same number of acquaintances within the group.

6. Give an inductive definition of concatenation of two strings.

7. Show that $(w^R)^R = w$ for all strings w.

8. Let $\Sigma = \{a, b\}$. Give examples of strings that belong to the following sets:

   a) $\{w \mid \text{for some } u \in \Sigma\Sigma, w = uu^Ru\}$
   b) $\{w \mid ww = www\}$
   c) $\{w \mid \text{for some } u, v \in \Sigma^*, uvw = wvu\}$
   d) $\{w \mid \text{for some } u \in \Sigma^*, www = uu\}$