T-79.144 Logic in Computer Science: Foundations

Examination, May 11, 2005

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).

(a) True or false: a proof method M is sound, if every valid sentence ϕ is provable using the method M.

(b) True or false: a conjunctive normal form ϕ of a sentence in predicate logic is logically equivalent to the form ϕ' obtained from ϕ by Skolemization.

(c) True or false: a sentence ϕ has at most as many *subsentences* as it has atomic sentences and connectives $(\neg, \land, \lor, \rightarrow, \leftrightarrow)$.

(d) True or false: if $\Sigma \not\models \phi$, then $\Sigma \models \neg\phi$.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \lor B \rightarrow C) \rightarrow \neg((A \land \neg C) \lor \neg(B \rightarrow C))$

(b) $\{\forall x \forall y (R(x,y) \rightarrow R(y,x))\} \models \forall x R(a,x)$

(c) $\{\forall x (P(x) \rightarrow Q(x) \lor R(x)), \neg \exists x R(x)\} \models \forall x (\neg Q(x) \rightarrow \neg P(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3

(a) Derive a clausal form for the sentence

$$\neg(\forall x P(x) \rightarrow \forall x \exists y Q(x,y)) \lor \neg \forall y P(y).$$

Try to make it as simple as possible.

(b) Consider the following program `P`:

```
v=0;z=0;while(!(z==y)){z=z+1;v=v-1};v=v+x
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{P}\ [\texttt{v==x-y}].$$

Assignment 4 Let us represent natural numbers $0, 1, 2, \ldots$ with ground terms 0, $s(0)$, $s(s(0))$,... built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x.

(a) Let the predicates $J2(x)$, $J3(x)$ and $J6(x)$ mean that a natural number x is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate $J6$ is based on the definitions of the predicates $J2$ and $J3$.

(b) Use resolution to show that if a natural number x is divisible by two and three, then the natural number $x + 6$ is divisible by six.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.