T-79.144 Logic in Computer Science: Foundations

Examination, December 17, 2004

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).

(a) True or false: the set $\{P(f(y), g(z, f(f(y)))), P(x, g(x,z))\}$ is unifiable.

(b) True or false: if Σ_1 and Σ_2 are sets of sentences such that $\Sigma_1 \subseteq \Sigma_2$ and ϕ is a sentence such that $\Sigma_1 \models \phi$, then also $\Sigma_2 \models \phi$.

(c) True or false: a proof method M is sound, if every valid sentence ϕ is provable using the method M.

(d) True or false: the satisfiability problem of propositional logic is **NP**-complete.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models \neg(A \wedge \neg B) \wedge (\neg C \rightarrow A) \rightarrow (A \wedge B) \vee (\neg A \wedge C)$

(b) $\{\forall x(P(x) \rightarrow Q(x)), \forall x(Q(x) \rightarrow R(x))\} \models \forall x(\neg P(x) \rightarrow \neg R(x))$

(c) $\{\forall x \neg(A(x) \leftrightarrow B(x)), \forall y A(y) \vee \forall y \neg A(y)\} \models \forall z B(z) \vee \forall z \neg B(z)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3

(a) Derive a clausal form for the sentence

$$\neg(\forall x P(x) \rightarrow \forall x \exists y Q(x,y)) \vee \neg \forall y P(y).$$

Try to make it as simple as possible.

(b) Consider the following program `P`:

```
v=0;z=0;while(!(z==y)){z=z+1;v=v-1};v=v+x
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{P}\ [\texttt{v==x-y}].$$

Assignment 4 A *directed* graph consists of a set of nodes connected by *directed* arcs. Let us assume that the nodes of the graph are named with constants $\{a, b, \ldots\}$ while the arcs of the graph are represented using a predicate $A(x,y)$ = "there is an arc from the node x to the node y".

(a) Define the predicates

$$\begin{array}{rcl} C(x,y) & = & \text{"there is a connection from the node } x \text{ to the node } y\text{"} \\ \text{and } L(x) & = & \text{"the graph has a loop that goes through the node } x\text{"} \end{array}$$

by taking the direction of arcs into account.

(b) Describe the directed graph given below using the predicate A. Use resolution to show that the sentence $\exists x(L(x) \wedge C(x,c))$ is a logical consequence of your description and the definitions of predicates C and L.

$$a \ \underset{\longleftarrow}{\overset{\longrightarrow}{}}\ b \longrightarrow c$$

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.

Feedback: `http://www.tcs.hut.fi/Studies/T-79.144/`