T-79.144 Logic in Computer Science: Foundations

Examination, October 4, 2004

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).

(a) True or false: it is possible to define the other propositional connectives ($\neg,\wedge,\vee,\leftrightarrow$) using the connectives $\rightarrow$ and $\underline{\vee}$ (exclusive or).

(b) True or false: if Σ_1 and Σ_2 are sets of sentences such that $\Sigma_1 \subseteq \Sigma_2$ and ϕ is a sentence such that $\Sigma_1 \models \phi$, then also $\Sigma_2 \models \phi$.

(c) True or false: a conjunctive normal form ϕ of a sentence in predicate logic is logically equivalent to the form ϕ' obtained from ϕ by Skolemization.

(d) True or false: the satisfiability problem SAT of propositional logic is **NP**-complete.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models \neg(A \wedge \neg B) \wedge (\neg C \rightarrow A) \rightarrow (A \wedge B) \vee (\neg A \wedge C)$

(b) $\{\exists x \exists y P(x,y), \forall x \forall y (P(x,y) \rightarrow Q(x,y))\} \models \exists x Q(x,x)$

(c) $\{\forall x \neg(A(x) \leftrightarrow B(x)), \forall y A(y) \vee \forall y \neg A(y)\} \models \forall z B(z) \vee \forall z \neg B(z)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3

(a) Derive a clausal form for the sentence

$$\neg(\neg \exists y E(y) \rightarrow \forall y (\exists x E(x) \rightarrow E(y))).$$

Try to make it as simple as possible.

(b) Consider the following program `P`:

```
z=0;v=x;while(!(z==y)){z=z+1;v=v-1}
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{P}\ [\texttt{v==x-y}].$$

Assignment 4 Let us represent natural numbers $0,1,2,\ldots$ with ground terms 0, $s(0)$, $s(s(0))$,... built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x+1$ for natural numbers x.

(a) Let the predicates $J2(x)$, $J3(x)$ and $J6(x)$ mean that a natural number x is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate $J6$ is based on the definitions of the predicates $J2$ and $J3$.

(b) Use resolution to show that if a natural number x is divisible by two and three, then the natural number $x+6$ is divisible by six.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.

Feedback: `http://www.tcs.hut.fi/Studies/T-79.144/feedback.html`