T-79.144 Logic in Computer Science: Foundations

Examination, October 6, 2003

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).

(a) True or false: propositional logic is decidable.

(b) True or false: the empty clause $\Box$ can be obtained from the clauses $\{P(x,y), P(y,x)\}$ and $\{\neg P(z,z), \neg P(w,w)\}$ by resolution.

(c) True or false: if a set of sentences $\Sigma \subseteq \mathcal{L}$ has exactly one model $\mathcal{A} \subseteq \mathcal{P}$, then it holds for each sentence $\phi \in \mathcal{L}$ that $\Sigma \models \phi$ or $\Sigma \models \neg\phi$.

(d) True or false: a sentence ϕ has at most as many *subsentences* as it has atomic sentences and connectives ($\neg$,$\wedge$,$\vee$,$\rightarrow$,$\leftrightarrow$).

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$

(b) $\{\forall x \exists y (P(x) \rightarrow Q(y)), \forall x P(x)\} \models \forall y Q(y)$

(c) $\{\forall x \forall y \forall z (R(x,y) \wedge R(y,z) \rightarrow R(x,z)), R(a,b), R(b,a)\} \models R(a,a)$

Tableau proofs must contain all intermediary steps !!!

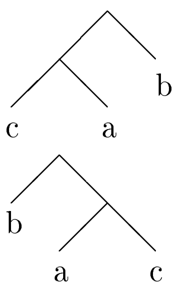
Assignment 3

(a) Derive a clausal form for the sentence $\neg(\forall x \forall y \neg B(y,x) \wedge \exists x (C(x) \rightarrow A(x)))$. Try to make it as simple as possible.

(b) Use a suitable invariant to establish that the function `min` below returns the least integer in a table `a` for which `size` > 0 holds.

```
int min(int a[], int size) {
   int m=a[0], i=1;
   while(i<size) { if(a[i]<m) m=a[i]; i=i+1; }
   return m;
}
```

Assignment 4 Binary trees are represented in terms of a binary function symbol i (inner nodes) and a unary function symbol l (leaf nodes). In this way, the upper tree in the picture gets a representation $i(i(l(c), l(a)), l(b))$.



(a) Let the predicate $\mathrm{M}(x,y)$ mean that binary tree x is the mirror image of binary tree y. Define the predicate M using sentences of predicate logic such that you can infer whether any given two binary trees are mirror images of each other (assuming the representation given above).

(b) Use semantic tableaux to show that the upper binary tree is the mirror image of the lower binary tree.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.

Feedback: `http://www.tcs.hut.fi/Studies/T-79.144/feedback.html`