T-79.144 Logic in Computer Science: Foundations

Examination, August 11, 2003

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).

(a) True or false: if $\Sigma \models \phi$ and $\models \phi \rightarrow \psi$, then $\Sigma \models \psi$.

(b) True or false: the empty clause $\Box$ can be obtained from the clauses $\{A, \neg B\}$ and $\{\neg A, B\}$ by resolution.

(c) True or false: Sheffer's stroke $|$ is definable using Peirce's arrow $\downarrow$.

(d) True or false: the satisfiability problem of propositional logic is **NP**-complete.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow C) \rightarrow (A \rightarrow B))$

(b) $\models (\forall x(P(x) \rightarrow \neg Q(x))) \rightarrow ((\exists x\, Q(x)) \rightarrow (\exists x\, \neg P(x)))$

(c) $\{\forall x(P(x) \rightarrow R(x)), \neg\exists x(\neg R(x) \wedge Q(x))\} \models \forall x(P(x) \vee Q(x) \rightarrow R(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3

(a) Derive a clausal form for the sentence $\neg(\forall x \forall y \neg B(y,x) \wedge \exists x(C(x) \rightarrow A(x)))$. Try to make it as simple as possible.

(b) Use a suitable invariant to establish that the function `min` below returns the least integer in a table `a` for which `size` > 0 holds.

```
int min(int a[], int size) {
   int m=a[0], i=1;
   while(i<size) { if(a[i]<m) m=a[i]; i=i+1; }
   return m;
}
```

Assignment 4 Let us represent natural numbers $0, 1, 2, \ldots$ with ground terms 0, $s(0)$, $s(s(0))$,... built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x.

(a) Let the predicates $J2(x)$, $J3(x)$ and $J6(x)$ mean that a natural number x is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate $J6$ is based on the definitions of the predicates $J2$ and $J3$.

(b) Use resolution to show that if a natural number x is divisible by two and three, then the natural number $x + 6$ is divisible by six.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.

Feedback: `http://www.tcs.hut.fi/Studies/T-79.144/feedback.html`