T-79.144 Logic in Computer Science: Foundations

Examination, February 10, 2003

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).

(a) True or false: the empty clause $\Box$ can be obtained from the clauses $\{A, \neg B\}$ and $\{\neg A, B\}$ by resolution.

(b) True or false: if $\models \phi$ and $\phi \wedge \neg\psi$ is unsatisfiable, then $\models \psi$.

(c) True or false: in propositional logic, at most 16 semantically different binary connectives can be defined.

(d) True or false: an existential quantifier can be moved outside an implication as follows: $(\exists x \phi(x) \rightarrow \psi)$ is rewritten as $\forall y(\phi(y) \rightarrow \psi)$ where y is a variable not appearing in formulas $\phi(x)$ and ψ.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models \neg(A \wedge \neg B) \wedge (\neg C \rightarrow A) \rightarrow (A \wedge B) \vee (\neg A \wedge C)$

(b) $\models \forall x(P(x) \wedge Q(x)) \leftrightarrow \neg(\exists y \neg P(y) \vee \exists z \neg Q(z))$

(c) $\{\forall x \forall y(R(x, y) \rightarrow R(y, x))\} \models \forall x R(a, x)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3

(a) Derive a clausal form for the sentence $\neg(\forall x \forall y(B(y) \rightarrow A(x, x)) \wedge \exists x(C(x) \rightarrow \forall y A(x, y)))$. Try to make it as simple as possible.

(b) Use a suitable invariant to establish that the function `min` below returns the least integer in a table `a` for which `size` > 0 holds.

```
int min(int a[], int size) {
   int m=a[0], i=1;
   while(i<size) { if(a[i]<m) m=a[i]; i=i+1; }
   return m;
}
```

Assignment 4 Let a ternary predicate $P(x, y, z)$ mean that the parents of a person x are y and z. Using this predicate, define the binary predicate $R(x, y)$ which means that x is a relative of y. Give a resolution proof that Kerttu is a relative of Kustaa using the following database in addition to your definition.

$$P(\text{kerttu}, \text{jaakoppi}, \text{hanna})$$
$$P(\text{jaakoppi}, \text{reino}, \text{lahja})$$
$$P(\text{kustaa}, \text{salme}, \text{reino})$$

Hint: relatives have an ancestor in common!

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.

Feedback: `http://www.tcs.hut.fi/Studies/T-79.144/feedback.html`