Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

T-79.144 Logic in Computer Science: Foundations

Examination, September 3, 2002

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: Sheffer's stroke $|$ is definable in terms of Peirce's arrow $\downarrow$.

(b) Does the following hold: if ϕ and ψ are different sentences in propositional logic, then their clausal forms are different, too.

(c) Does the following hold: if $\Sigma \not\models \phi$ and $\Gamma \subseteq \Sigma$, then $\Gamma \not\models \phi$ (Σ and Γ are sets of sentences).

(d) Does the following hold: predicate logic is decidable.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (\neg B \to \neg A) \to ((\neg B \to A) \to B)$

(b) $\{\forall x \forall y(R(x,y) \to R(y,x))\} \models \forall x \forall y(R(x,y) \vee R(y,x))$

(c) $\models \forall x(P(x) \leftrightarrow \neg Q(x)) \leftrightarrow \neg \exists x(P(x) \leftrightarrow Q(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Formalize the following claims in terms of predicate logic:

1. If a brick is on another brick, it is not on the table.
2. Every brick is on the table or on another brick.
3. No brick is on a brick which is also on some other brick.

---

4. If a brick is on another brick, then the latter brick is on the table.

Use resolution to show that the sentence 4 is a logical consequence of the sentences 1-3.

Assignment 4 A *directed* graph consists of a set of nodes connected by *directed* arcs. Assume that nodes are represented with constants $\{a, b, \ldots\}$ while arcs are represented with a binary predicate $A(x,y)$ = "there is an arc leading from the node x to the node y".

(a) Define the predicates

$C(x,y)$ = "there is a connection from the node x to the node y"
and $L(x)$ = "the graph has a loop that goes through the node x"

by taking the direction of arcs into account.

(b) Describe the directed graph below using the predicate A. Show that $\exists x \exists y(L(x) \wedge A(x,y) \wedge L(y))$ is a logical consequence of your description and the definitions of predicates C and L. Use semantic tableaux.

$$a \underset{\longleftarrow}{\overset{\longrightarrow}{}} b \longrightarrow c$$

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.