

**Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!**

Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ  
T-79.144 Logic in Computer Science: Foundations  
Examination, May 13, 2002

**Assignment 1** Answer and justify briefly, but exactly.

- (a) Does the following hold: if  $\Sigma \cup \{\neg\phi\} \models \phi \wedge \neg\phi$  holds for a set of sentences  $\Sigma$  and a sentence  $\phi$ , then  $\Sigma \models \phi$ .
- (b) Does the following hold: the empty clause  $\square$  can be obtained from the clauses  $\{P(x), P(y)\}$  and  $\{\neg P(z), \neg P(w)\}$  by resolution.
- (c) Does the following hold: the connective  $\neg$  is definable using the connective  $\rightarrow$ .
- (d) Does the following hold: propositional logic is decidable.

**Assignment 2** Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

- (a)  $\{B \rightarrow A, C \rightarrow B, \neg D \rightarrow \neg(C \rightarrow A)\} \models D$
- (b)  $\{\exists x \exists y P(x, y), \forall x \forall y (P(x, y) \rightarrow Q(x, y))\} \models \exists x Q(x, x)$
- (c)  $\{\forall x (P(x) \rightarrow R(x)), \neg \exists x (\neg R(x) \wedge Q(x))\} \models \forall x (P(x) \vee Q(x) \rightarrow R(x))$

Tableau proofs must contain all intermediary steps !!!

**Assignment 3** Consider a binary predicate  $R$  which is interpreted as a binary relation  $R^A \subseteq A \times A$  with respect to a universe  $A$ .

- (a) Give sentences of predicate logic that define when  $R^A$  is (1) *reflexive*, (2) *symmetric*, (3) *transitive* and (4) an *equivalence relation*.
- (b) Use resolution to establish that  $R^A$  is an equivalence relation, if it is symmetric, transitive and *serial* (as defined by  $\forall x \exists y R(x, y)$ ).
- (c) Use resolution to examine if equivalence relations are always serial.

**Assignment 4** Formalize the following claims in terms of predicate logic:

1. Alders are leaf trees.
2. Trees are spruces, alders or pines.
3. Spruces and pines are conifer trees.
4. Trees are leaf trees or conifer trees.

Use semantic tableaux to show that the sentence 4 is a logical consequence of the sentences 1-3.