

**Assignment 1** Answer and justify briefly, but exactly.

- (a) Does the following hold: propositional logic is decidable.
- (b) Does the following hold: the empty clause  $\square$  can be obtained from the clauses  $\{P(x, y), P(y, x)\}$  and  $\{\neg P(z, z), \neg P(w, w)\}$  by resolution.
- (c) Does the following hold: if a set of sentences  $\Sigma$  has at most one model, then it holds for each sentence  $\phi$  that  $\Sigma \models \phi$  or  $\Sigma \models \neg\phi$ .
- (d) Does the following hold: a sentence  $\phi$  has at most as many *subsences* as it has atomic sentences and connectives ( $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$ ).

**Assignment 2** Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

- (a)  $\models (\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B)$
- (b)  $\{\forall x \exists y (P(x) \rightarrow Q(y)), \forall x P(x)\} \models \forall y Q(y)$
- (c)  $\{\forall x (A(x) \leftrightarrow \neg B(x)), \forall x (B(x) \leftrightarrow \neg C(x)), \forall x (C(x) \leftrightarrow \neg A(x))\} \models \forall x (A(x) \wedge B(x) \wedge C(x))$

Tableau proofs must contain all intermediary steps !!!

**Assignment 3** Show that the sentence

$$\exists x (R(x) \wedge \neg R(f(f(x)))) \rightarrow \exists x (R(x) \wedge \neg R(f(x)))$$

is valid by linear resolution.

**Assignment 4** Let the predicate  $M(x, y)$  mean that a person  $x$  has met a person  $y$ , and the predicate  $S(x, y)$  that a person  $x$  is sick of an infectious disease  $y$ .

- (a) Use the predicates given above to define a predicate  $D(x, y)$  which means that a person  $x$  is in danger of infection by a disease  $y$ .
- (b) Consider the following database.

$$\begin{array}{ll} M(\text{valentine, ronald}) & S(\text{valentine, flu}) \\ M(\text{lilian, ronald}) & S(\text{harry, chickenpox}) \vee S(\text{ronald, rubeola}) \\ M(\text{harry, lilian}) & \end{array}$$

Use semantic tableaux to show that somebody is in danger of infection by flu and chickenpox, *or* by flu and rubeola. The tableaux proof must be based on the database and the definition of the preidcate  $D(x, y)$ .