

Assignment 1 Answer and justify briefly, but exactly.

- (a) Does the following hold: the empty clause \square can be obtained from the clauses $\{P(f(y, g(y)))\}$ and $\{\neg P(f(g(x), g(g(x))))\}$ by resolution.
- (b) Does the following hold: if $\Sigma \models \phi$ and $\Sigma \models \neg\phi$ for some sentence ϕ , then the set of sentences Σ is unsatisfiable.
- (c) Does the following hold: if θ and θ' are two most general unifiers of a set of atomic formulas S , then $\theta = \theta'$.
- (d) Does the following hold: it is possible to define the other propositional connectives ($\wedge, \vee, \leftrightarrow$) using the connectives \rightarrow and $\underline{\vee}$ (exclusive or).

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

- (a) $\{A \leftrightarrow \neg B, B \leftrightarrow \neg C, C \leftrightarrow \neg A\} \models A \wedge B \wedge C$
- (b) $\{\forall x \exists y P(x, y)\} \models \exists x \forall y P(x, y)$
- (c) $\{\forall x \forall y (\exists z (R(x, z) \wedge R(z, y)) \rightarrow R(x, y)), R(a, b), R(b, a)\} \models R(a, a)$.

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Natural numbers $0, 1, 2, \dots$ are represented as ground terms $0, s(0), s(s(0)), \dots$ built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x .

- (a) Let the predicates $J2(x)$, $J3(x)$ and $J6(x)$ mean that a natural number x is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate $J6$ is based on the definitions of the predicates $J2$ and $J3$.
- (b) Use resolution to show that if a natural number n is divisible by two and three, then the natural number $n + 6$ is divisible by six.

Assignment 4 Formalize the following claims in terms of predicate logic:

1. Alders are leaf trees.
2. Trees are spruces, alders or pines.
3. Spruces and pines are conifer trees.
4. Trees are leaf trees or conifer trees.

Use semantic tableaux to show that the sentence 4 is a logical consequence of the sentences 1-3.