T-79.1001 **Syksy 2006**
Introduction to Theoretical Computer Science (T)
Session 6
Answers to demonstration exercises

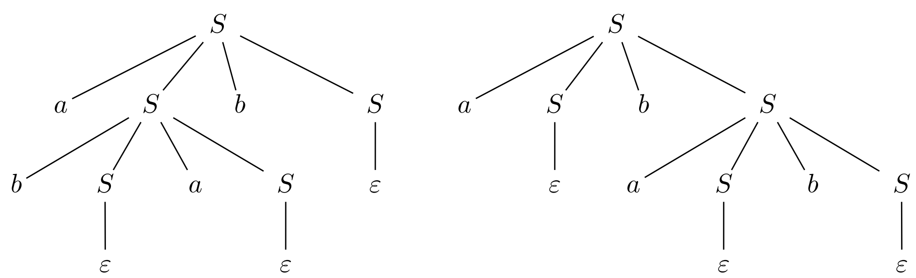
4. **Problem**: Construct a context-free grammar for the language $\{w \in \{a,b\}^* \mid w$ has as many as as bs$\}$.

   Solution: There are several different ways of designing a grammar for this language. The simplest answer is the ambiguous grammar:

$$S \to aSbS \mid bSaS \mid \varepsilon \ .$$

   The first rule of the grammar expresses the condition: "If the string starts with an a, then at some point of the string there has to be a corresponding b. Between these two symbols there may be arbitrary balanced strings."

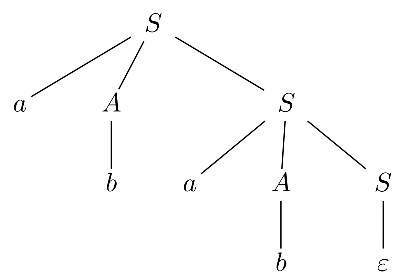
   For example, the string $abab$ has two parse trees:



   If we want to have an unambiguous grammar for the language, we have to ensure that the first a is associated with the first possible b:

$$\begin{aligned} S &\to aAS \mid bBS \mid \varepsilon \\ A &\to aAb \mid b \\ B &\to bBb \mid a \end{aligned}$$

   Now $abab$ has only one parse tree:



5. **Problem**: Prove that the following context-free grammar is ambiguous:

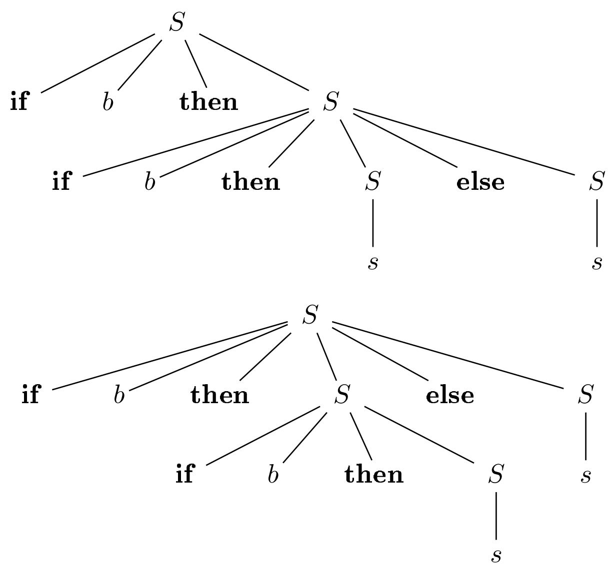
$$\begin{aligned} S &\to \textbf{if } b \textbf{ then } S \\ S &\to \textbf{if } b \textbf{ then } S \textbf{ else } S \\ S &\to s. \end{aligned}$$

   Design an unambiguous grammar that is equivalent to the grammar, i.e. one that generates the same language.

Solution: A context-free grammar is ambiguous if there exists a word $w \in L(G)$ such that w has at least two different parse trees. The simplest word for the given grammar that has this property is:

$$\textbf{if } b \textbf{ then if } b \textbf{ then } s \textbf{ else } s.$$

Its two parse trees are:



Usually we want to associate an **else**-branch to the closest preceeding **if**-statement. In this case the former tree corresponds to this practice.

We define a grammar G as follows:

$$\begin{aligned}
G &= (V, \Sigma, P, S) \\
V &= \{S, B, U, s, b, \textbf{if}, \textbf{then}, \textbf{else}\} \\
\Sigma &= \{s, b, \textbf{if}, \textbf{then}, \textbf{else}\} \\
P &= \{S \to B \mid U \\
&\quad\; B \to \textbf{if } b \textbf{ then } B \textbf{ else } B \;\mid s \\
&\quad\; U \to \textbf{if } b \textbf{ then } S \mid \textbf{if } b \textbf{ then } B \textbf{ else } U\}
\end{aligned}$$

Here the nonterminal B is used to derive balanced programs where each **if**-statement has both **then**- and **else**-branches. The nonterminal U derives those **if**-statements that do not have an **else**-branch.

6. **Problem**: Design a recursive-descent (top-down) parser for the grammar from Problem 6/6.

   Solution: The following C-program implements a top-down parser for the following grammar:

$$\begin{aligned}
C &\to S \mid S; C \\
S &\to a \mid \textbf{begin } C \textbf{ end} \mid \textbf{for } n \textbf{ times do } S
\end{aligned}$$

   This grammar is a simplified form of the one in problem 6.6. The difference is that all different numbers are replaced by a new terminal symbol n that denotes a number.

   The most important functions of the program are:

   - `C()`, `S()` — implement the rules of the program.

- `lex()` — read the next lexeme from the input, and store it in a global variable `current_tok`.
- `expect(int token)` — tries to read the lexeme *token* from input. Gives an error message if it fails.
- `consume_token()` — mark the current lexeme used. This is necessary because sometimes we have to have a one-token lookahead before we know what rule we must apply.

In practice, the programming language parsers are implemented using *lex* and *yacc* tools[1]. Of these, *lex* generates a finite automaton-based lexical analyser from identifying lexemes that have been defined using regular expression, and *yacc* constructs a pushdown automaton-based parser for a given context-free grammar.

```
#include <stdio.h>
#include <stdlib.h>
#include <ctype.h>

/* Define the alphabet */
enum TOKEN  { DO, FOR, END, BEGIN, TIMES, OP, SC, NUMBER, ERROR };
const char* tokens[] = { "do", "for", "end", "begin", "times", "a",
                         ";", "NUMBER", NULL };

/* A global variable holding the current token */
int current_tok = ERROR;

/* Maximum length of a token */
#define TOKEN_LEN 128

/* declare functions corresponding to nonterminals */
void S(void);
void C(void);


int lex(void);
void consume_token(void);
void error(char *st);
void expect(int token);

void C(void)
{
  S();
  lex();
  if (current_tok == SC) {
    consume_token();
    C();
    printf("C => S ; C\n");
  } else {
    printf("C => S\n");
  }
}

void S(void)
{
```

[1] Or some of their derivatives, like *flex* or *bison*.

```
  lex();
  switch (current_tok) {
  case OP:
    consume_token();
    printf("S => a\n");
    break;
  case BEGIN:
    consume_token();
    C();
    expect(END);
    printf("S => begin C end\n");
    break;
  case FOR:
    consume_token();
    expect(NUMBER);
    expect(TIMES);
    expect(DO);
    S();
    printf("S => for N times do S\n");
    break;
  default:
    error("Parse error");
  }
}

/* int lex(void) returns the next token of the input. */
int lex(void)
{
  static char token_text[TOKEN_LEN];
  int pos = 0, c, i, next_token = ERROR;

  /* Is there an existing token already? */
  if (current_tok != ERROR)
    return current_tok;

  /* skip whitespace */
  do {
    c = getchar();
  } while (c != EOF && isspace(c));
  if (c != EOF) ungetc(c, stdin);

  /* read token */
  c = getchar();
  while (c != EOF && c != ';' && !isspace(c) && pos < TOKEN_LEN) {
    token_text[pos++] = c;
    c = getchar();
  }
  if (c == ';') {
    if (pos == 0) /* semicolon as token */
      next_token = SC;
    else { /* trailing semicolon, leave it for future */
      ungetc(';', stdin);
    }
  }
```

```
  token_text[pos] = '\0'; /* trailing zero */

  /* identify token */
  if (isdigit(token_text[0])) { /* number? */
    next_token = NUMBER;
  } else { /* not a number */
    for (i = DO; i < NUMBER; i++) {
      if (!strcmp(tokens[i], token_text)) {
        next_token = i;
        break;
      }
    }
  }
  current_tok = next_token;
  return next_token;
}

void consume_token(void)
{
  current_tok = ERROR;
}

void error(char *st)
{
  printf(st);
  exit(1);
}

/* try to read a 'token' from input */
void expect(int token)
{
  int next_tok = lex();
  if (next_tok == token) {
    consume_token();
    return;
  } else
    error("Parse error");
}

int main(void)
{
  int i;
  C();
  return 0;
}
```